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The Importance of Teaching and Learning Patterning in Early Math Education

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Abstract

This literature review explores the importance of patterning in early math education. Patterning skills are a unique predictor of future success in mathematics. With the Common Core not addressing patterning as a standard in early math education, many educators have come to believe that the importance of patterning is not significant. However, past and present research studies have yielded convincing results that demonstrate the importance of including patterning in early math curriculums. Incorporating patterning into young students' math experiences requires several specific instructional strategies to optimize the benefits of learning patterning skills.

The Importance of Teaching and Learning Patterning in Early Math Education

Patterns exist in all aspects of a young child's world, whether it be the stripes on a zebra, the repetition of shapes on a building's facade, or the repeating steps of a dance. Patterning is the ability to define the rule(s) in a predictable sequence of items (Pasnak, 2017). Research has determined that a child's skill in patterning is one of the best indicators of future success in mathematics (Rittle-Johnson, et al., 2019). If a child is not proficient in his/her ability to recognize patterns and determine the rule guiding the pattern, it is likely that algebraic reasoning will be difficult (Lee, 2011). Since patterning is a crucial component of pre-algebraic thinking the National Council of Teaching Mathematics (NCTM) has determined that algebra is not a stand-alone course, but rather the culmination of experiences that lend to developing algebra skills. Patterning is related to algebraic thinking because it allows children to notice similarities and differences then create rules that govern the pattern. When patterning is presented in early math education, children are more likely to develop the skills necessary to understand the relationships within a pattern and eventually use symbols to represent those relationships (McGarvey, 2013). The problem is that even though research has determined the importance of patterning and its relationship with future mathematical achievement, it is often not considered an integral part of students' math development and therefore not taught as extensively as it needs to be.

The purpose of this literature review is to demonstrate the importance of teaching and learning patterning in early math education. Current research has discovered that mathematical skills in the upper grades are linked to what is learned in early math education and that patterning plays an important role in developing these skills. Research into patterning has also yielded information as to what strategies and best practices are foremost for teaching and learning

patterning skills. Finally, evidence from numerous studies has proven why patterning should be included in early math education curriculums.

Research included in this review is relevant, and the majority of studies have been completed within the past ten years, therefore adding to the significance of patterning in the current educational system. All articles hail from scholarly journals that have been peer-reviewed and contain research that is relevant to early learning or can be applied to concepts related to early learning. Additional information has been obtained from respected organizations such as the

National Association for the Education of Young Children (NAEYC), Iowa Early Learning Standards, National Council of Teachers of Mathematics, and various other educational organizations. In order for articles to be included in this review, they must relate to the concept skill of patterning. Although some articles also include topics such as spatial awareness or shapes, the researcher specifically focused on the data related to patterning. Articles dated prior to 2010 were not included unless they included information that spoke to the historical aspect of patterning research.

This literature review seeks to prove the thesis that the teaching and learning of patterning is a critical and necessary component of any early learning mathematics curriculum. When patterning is taught as part of an early learning curriculum and built upon with advanced skill development throughout the primary years, students have the opportunity to acquire the cognitive skills needed to increase future academic achievement in mathematics. Furthermore, when students learn instructional strategies related to creating and expanding patterns, they are also better able to relate those strategies to other areas of mathematics, such as algebra.

The following literature review is four-fold. First, it will briefly examine the historical ramifications of patterning to set the premise for today's research studies. Secondly, it will introduce numerous research studies that support the importance and positive effects of teaching patterning as part of an early math education curriculum. The third aspect of this paper will focus on the significance of learning patterning skills and provide research-based instructional strategies that can be implemented to increase patterning skills. The paper will conclude with a summary of evidence to support the thesis of this literature review.

Literature Review

Historical Perspective

In 1968, the president of the National Council of Teachers of Mathematics, Donovan Johnson (1968) authored an article in which he compared mathematics to a musical composition because the logic and patterns of mathematics fit together like the pieces of a song. He noted that patterns in mathematics, such as the pattern that exists with multiples of nine, engages students while allowing them to practice number operations. Over the year's researchers have focused on numerous components of patterning in math education. One notable early researcher was Lynn Steen (1988), who described patterns as the language and science of mathematics because one is constantly searching for patterns not only in mathematics but also in the world. Johnson noted the importance of finding patterns in nature and cited the honeycomb as a brilliant example: "Mathematics is an exploratory science that seeks to understand every kind of pattern; patterns that occur in nature, patterns invented by the human mind, and even patterns created by other patterns. To grow mathematically, children must be exposed to a rich variety of patterns appropriate to their own lives through which they can see variety, regularity, and interconnections" (Steen, 1988, p. 611).

Patterning has been a topic of interest in the world of mathematics for many years. Illinois State University mathematics professor Carol Thorton (1977) noted that children begin to notice patterns early in life and that recognizing patterns is an important foundational skill that is necessary to the development of math skills and concepts. Burton (1982) echoed those thoughts and believed that recognizing the similarities and differences that occur in patterning is essential to success in mathematics.

Papic & Mulligan (2007) conducted one of the earliest pieces of research that focused specifically on patterning in early learning. This qualitative research took place within two matched preschools that were similar in staffing, demographics, programming, and parent involvement. One preschool received interventions related to patterning while the other preschool maintained its regular curriculum without any patterning interventions. The study utilized individual interviews to assess each student's ability to complete various patterning skills such as designing a pattern, copying a pattern, or continuing a pattern. Students were interviewed initially in June and then again in December. The students in the non-intervention school scored higher during the first assessment; however, during the December qualitative interview the preschool students who had participated in the intervention group scored significantly higher than the students who did not receive the patterning intervention.

Although this was not a controlled study, the results of this early research have provided valuable information with regards to the study of patterning in early childhood classrooms. The researchers' data determined that the students who had the additional interventions in patterning showed improvement over the course of the six months with regards to their ability to complete patterning tasks. The researchers also noted that the growth was seen twelve months later, although this data was not included in their study. The study drew the conclusion that patterning

is important not only to abstract thinking but also to the representation of mathematical concepts. The researchers found that students who had done poorly with patterning in the study showed lower achievement of numeracy skills at the end of their first year of school, thus demonstrating a connection between a child's patterning skills and their ability to reason. They believed that the ability to see similarities and differences in patterning were important for pre-algebraic thinking and determined that patterning has an impact on future learning (Papic & Mulligan, 2007). However, Schmerold et al. (2016) noted that it is possible that since an analysis of the statistics was not completed, the abilities of the control group could have been inferior to the intervention group with regards to skill levels at the time of the study.

Mulligan & Mitchelmore (2009) conducted a research study to investigate the correlation between an awareness of mathematical patterning and structure (AMPS) and varying concept areas in mathematics in early mathematical learning. They theorized that students who are able to notice similarities and differences in patterns are more likely to understand the structure of patterns and apply this knowledge to other mathematical concepts and processes. The quantitative study was conducted on 103 first grade students and utilized an assessment they developed entitled Pattern and Structure Assessment (PASA), which consisted of thirty-nine tasks. These tasks were fundamental math concepts that centered around number, measurement, and space. The PASA scores ranged from 3 to 33 out of a possible 39, and the scores were considered consistent with mathematical skills of first graders and therefore deemed valid and reliable.

Mulligan and Mitchelmore (2009) concluded that having a greater level of pattern awareness correlates to a higher level of achievement in mathematics. The results of this early study brought attention to how children develop mathematically, especially with regards to

algebraic thinking. Papic et al. (2007) states that often the teaching of pre-algebraic concepts to early learners is considered developmentally inappropriate; however, this study suggests that is not the case. The difficulties older students have can be linked to limited experiences in mathematics in the early years. These early findings are significant because they set the foundation for a connection between patterning and mathematical achievement.

Significance of Teaching Patterning

Patterns are considered to play an important role in early mathematics education (Tsamir et al. 2017) and are typically introduced in early childhood classrooms. Within the past ten years researchers have undertaken the study of patterning and how it is linked to working memory, executive function, fluid reasoning, spatial skills, and reading in addition to mathematical achievement. While this research is important, this literature review will narrow the focus to conclusions that speak to the effect of patterning on mathematical skills and achievement.

Patterning skills are taught as part of early childhood math curriculums because most educators feel patterning has value (Kidd et al., 2013). The Iowa Early Learning Standards (2018) states that children should develop an understanding of patterns and be able to recognize, create, and extend both simple and complex patterns in a variety of settings, including nature. In contrast, the Common Core (2010) does not include any standards that are related to patterning in grades kindergarten through second grade. The Common Core mentions numerical patterning as a standard in grades three through five and again briefly as a math practice skill. Math Practice Skill MP.7 states that patterning is a skill that should be developed, and students should look closely to ascertain if a pattern exists. The omission of patterning as a Common Core standard in kindergarten through second grade was based on insufficient research evidence to support the inclusion of patterning at the time the standards were written (Rittle- Johnson et

al., 2019). Many educators support the implementation of patterning in early mathematics education and the inclusion of patterning as a specific standard within the Common Core (Zippert et al., 2020). Although not all researchers see the merit in teaching patterning (Burgoyne et al., 2017), recent studies have garnered evidence that connects patterning and mathematical achievement.

The contradiction of the importance of teaching patterning skills in recent years has led researchers to study patterning in order to ascertain the role patterning plays with learning mathematical concepts and how patterning affects achievement. Research studies seek to answer the question of whether understanding patterning leads to an increase in mathematical academic performance (Kidd et al., 2013). Current research findings contribute to the idea that patterning is connected with mathematics knowledge (Fyfe et al., 2017). As a result, researchers have concluded that patterning and its effect on mathematical knowledge warrants additional study to ascertain the relationship between the two. Recent studies have yielded information not only as to how teaching patterning relates to future mathematical achievement, but also to specific mathematical achievement such as numeracy, calculations, and algebra (Zippert et al., 2020).

Patterning and Mathematical Skills

Researchers have determined that when students are taught patterning skills, they are better able to complete patterning tasks than those students who were not instructed in patterning (Kidd et al., 2013). In addition, when young learners are able to compare two different patterns that are made with the same materials, the result is a higher level of reasoning (Tsamir et al., 2017). A long-term study by Papic, Mulligan and Mitchelmore (2011) compared two groups of preschoolers. Over the course of six months one group received a pattern-based intervention while the comparison group did not. The intervention group outperformed the non-intervention

group in patterning skills at the end of the study. More importantly, the students who received the patterning intervention continued to do better with repeating patterns one year later in kindergarten and again in first grade, when compared to the non-intervention group. In addition, the researchers reported that the students in the intervention group were able to successfully complete tasks that were not part of the intervention and performed better on an assessment that measured numeracy, which is the understanding of whole numbers and included counting.

Patterning instruction can have a positive effect not only on patterning skills, but in mathematics as well (Schmerold et. al., 2016). Kidd et al. (2013) divided first graders into four intervention groups for an additional fifteen minutes of direct instruction each day in one of four areas: patterning, reading, math, and social studies. The group that received the additional patterning instructions improved their patterning scores when compared to the other three groups. The patterning intervention group also scored higher than the other groups on an assessment that measured mathematical concepts. Consequently, the researchers concluded that academic achievement is bolstered by patterning instruction.

In addition to advancing patterning skills, patterning instruction has been shown to have an effect on the development of numeracy skills. Students who receive patterning instruction performed at a higher level on an assessment that measured numeracy than those who did not receive patterning interventions (Papic et al., 2011). Rittle-Johnson et al. (2019) noted that a child's ability to pattern is a good indicator of general math knowledge and numeracy skills. Although students do not need to have a knowledge of numbers to complete patterning tasks, patterning was a predictor of numeracy knowledge and educators should use patterns as a method for promoting numeracy.

Not all researchers agree that patterning ability is the best indicator of mathematical achievement. Nguyen et al. (2016) conducted a study to ascertain what skills were important for future mathematical success. The study evaluated preschoolers in the fall and spring on counting, number recognition, patterns, measurement, and geometry. A similar test geared for third through fifth graders was given to measure the proficiency of the math skills of fifth grade students. The study concluded that although geometry, patterning, and measurement were predictive of mathematical achievement, counting and numeracy skills were considered to be a better indicator of mathematical achievement in fifth grade.

Pattern knowledge in preschool has been shown to predict general math knowledge and numeracy skills in kindergarten, specifically the ability to count to 100. Pre-kindergarten students with higher repeating patterning scores were 27 times more likely to be able to count to 100 by the time they completed kindergarten than their peers who were not as proficient in completing repeating patterns (Zippert et al., 2020).

Researchers have concluded that patterning knowledge influences numeracy skills because numbers follow patterns (Zippert et al., 2020). Numeracy is enhanced by patterning due to the predictable sequence found in numbers: when students learn to find patterns, counting and calculation skills are enhanced (Fyfe et al., 2017). Patterning promotes additional counting skills such as skip counting, especially by twos and fives, since counting sequences contain the numerals zero to nine and the numbers repeat over again with each decade thus creating a pattern (Rittle-Johnson et al., 2019).

In addition, patterning skills have been found to be predictive of calculations skills, such as addition and subtraction (Fyfe et al., 2017). Early learners who are able to complete complex repeating patterns demonstrate advanced knowledge of addition and subtraction computation

skills at the end of first grade. In addition, the ability to repeat patterns abilities yielded an understanding of addition and subtraction strategies other than counting (Luken et al., 2014). Patterning is connected to calculation skill because of the predictable sequences that students develop over time while identifying and describing sequences of numbers. Examples of predictable sequences include adding two even numbers together to produce an even number, adding nine to a number results in an answer that is ten more than the given number minus one, and the next number in a sequence is one plus the previous number (Fyfe et al., 2017).

Another key area of research with regards to patterning is the effect of pattern instruction on algebraic skills. Algebraic thinking involves generalization, which is the idea that there is a consistent relationship that remains the same (Papic et al., 2011) Prealgebraic thinking is developed when students search for the relationship between random objects or symbols (Pasnik et al., 2019). Lee et al. (2011) researched the effects of patterning on algebraic word problems with nine- and ten-year-old students in Singapore. The study hypothesized that patterning skills influenced algebraic skills. The results indicated that students who are proficient with number patterns did better with algebraic reasoning most likely due to the idea that patterns and algebra utilize the same thought processes. The evidence compiled in the study was able to show a connection between patterns and algebraic skill; however, the study was not able to ascertain which components of patterning were most important for algebraic problem solving. However, the study also concluded that although patterning plays a significant role in algebraic proficiency, updating and computational abilities played a greater role in predicting algebraic performance.

Analyzing the structure of repeating patterns leads early learners to think in terms of how items are related, thus increasing early algebra skills (Plessis, 2018). Learning to recognize the rules and relationships that lie within a pattern is especially important with regards to algebraic

reasoning. Young learners who learn and apply rules when given a new pattern demonstrate an understanding of how the parts of a particular pattern are related; this knowledge forms a foundation for algebraic reasoning (Gadzichowski, 2012).

Patterning and Mathematical Achievement

Patterning positively predicts general math knowledge (Rittle-Johnson et al., 2019). Mulligan, Oslington, and English (2020) concluded that students who received an intervention in patterning demonstrated significant growth in awareness of pattern and structure when compared to students who did not receive the additional intervention. When the groups were compared one year later, the difference between the two groups was even greater. The impact of the patterning instruction had a positive influence on the first-grade skills of the intervention group, and their teachers reported that the intervention students were able to explain and make connections between mathematical concepts. The study did note that this intervention was more intense and encompassed more classroom time then a typical classroom mathematics instruction. An unexpected key finding was that students were able to develop patterning and structure skills that were above the normal curriculum expectations in kindergarten and appeared to support early algebraic thinking.

Rittle-Johnson et al. (2017) concluded that early knowledge of patterning elicited higher math achievement several years later. The researchers synthesized that working with patterns helps students learn to identify underlying rules and develops an awareness of spatial skills. Although both studies concurred on the importance of patterning to future mathematical achievement, Rittle-Johnson et al.'s study found patterning to be the most influential factor, not numeracy or counting.

Schmerold et al. (2016) researched the correlation between patterns, executive function, and mathematics with first graders, finding a significant relationship between patterning and mathematical achievement. Patterning skills were more highly related to mathematical achievement than to executive function. Patterning skills are predictive of math skills (Zippert et al., 2020). This study suggest that older students would also benefit from patterning instruction and that more complex patterns result in more progress in mathematical skills.

Research has determined that patterning does play a predictive role in current and future mathematical achievement (Rittle-Johnson et al., 2019). The importance of patterning in early learning is important because patterns lead to the discovery and understanding of concepts in mathematics (Kandir et al., 2018). The impact that patterning has on mathematical skills such as numeracy (Zippert et al., 2020), calculations (Fyfe et al., 2017) and algebra (Plessis et al., 2018) has led researchers to conclude that patterning does have a positive impact on future mathematics performance (Rittle-Johnson et al., 2017). Patterning skills have emerged as a quintessential component of math education and research needs to continue to focus on its relationship to mathematics, especially the nature of how patterns are understood, and what training is needed in order for patterning skills transfer to mathematical knowledge (Burgoyne et al., 2019) and to clarify the role patterning plays in regard to a student's skill (Rittle-Johnson et al., 2019) The next step is to examine how students best learn patterning skills in order to maximize the positive effect patterning has on future mathematics achievement (Tsamir et al., 2017).

Significance of Learning Patterning

Since research has shown a connection between early knowledge in patterning and later mathematical achievement, the next step is to ascertain which methods or strategies prove most beneficial for increasing future mathematical skills (Collins & Laski, 2015). It is vitally

important that teachers understand how teaching strategies should be used in early math education and which are best for promoting patterning skills that lead to future mathematical achievement (Clements & Sarama, 2011). In order to determine these skills, one must first understand what constitutes a pattern. Papic, Mulligan and Mitchelmore (2011) stated that the structure of a pattern is a basic repetition that incorporates various features. The general consensus within math education is that the key component of pattern is that of repetition (McGarvey, 2013).

Patterns are often described by using abstract labels such as the letters of the alphabet with each letter representing a different component of the pattern, for example ABAB; patterning is the ability to recognize and maneuver these patterns (Schmerold et al., 2016). Patterns are predictable and have a structure or rule that govern them (McGarvey, 2013); however, research has determined that the dimension by which the pattern is presented does not influence the result (Gadzichowski, 2012).

Patterns

The structure of patterns can vary not only in terms of attributes, but in complexity as well. The most common type of patterning skills revolves around students manipulating patterns either by copying, extending, or creating. Research has shown that copying a pattern is the easiest skill for students with 71% of kindergarten students able to complete this task. Expanding a pattern was seen as more difficult with a 48% success rate. Creating a new color pattern found 21% of students successful while over 45% were unable to create even the most basic pattern (Reid & Andrews, 2016). There are three types of patterns that are significant in early mathematics: repeating patterns, spatial structural patterns, and growing patterns (Papic et al., 2011).

Repeating patterns are patterns that have a recognizable unit of repeat that occurs continuously and contains more than one variable (Hutchinson, 2011). Repeating patterns are often the first types of patterns that early childhood students encounter (Collins & Laski, 2015) and are the easiest type of pattern for early learners to master (Gadzichowski, 2012). The simplest version of this type of pattern involves two alternating items and is typically represented as ABAB (Papic et al., 2011). Visual repeating patterns do not involve requiring the student to have any additional math skills such as numeracy or operations (Collins & Laski, 2015). Repetitive patterns provide a sense of organization to a child when items do not seem to be able to be classified (Bock et al., 2018).

Spatial structural patterns are typically composed of geometrical shapes or other mathematical shapes such as grids or blocks. The pattern may include features such as spacing, size, or number (Papic et al., 2011). These patterns are organized in such a way that they are easily recognizable; for example, the pattern could consist of stars within a rectangle (Hutchinson & Pournara, 2011).

Growing patterns are characterized by items that ascend or descend in a systematic manner, such as numbers that increase or decrease as the pattern progresses (Bock et al., 2018). Growing patterns can be spatial patterns (Papic et al., 2011) since these types of patterns are also frequently associated with patterns of spatial structures, such as shapes that increase in size (Hutchison & Pournara, 2011). Growing patterns can incorporate different groups of shapes, objects, or numbers (Geist et al., 2012) and are considered important to the development of skip counting because they emphasize the importance of increases and decreases (Gadzichowski, 2012). It is important that teachers incorporate many different types of patterns into their

mathematics curriculum (Papic et al., 2011) as patterning instruction must advance beyond alternating repeating patterns (Gadzichowski, 2012).

Expanding beyond basic patterns to more complex patterns is crucial to developing mathematical knowledge. A fundamental aspect of mathematics is understanding systematic increases, so providing opportunities for students to work on growing patterns is very important. A study of first graders determined that students do not necessarily expand their understanding to advanced patterns, especially those which consist of more elements, so instruction on more complex patterns will prove to be very beneficial (Gadzichowski, 2012).

The structure of each type of pattern can vary. A pattern can have a repeating unit such as alternating colors (e.g. red-green-red-green) or a repeating rule such as adding two to the previous numeral (e.g. 2-4-6-8). Patterns can also be composed of a growing relationship as in the example of doubling the last number (e.g.1-2-4-8-16). Patterns can utilize colors, shapes, sizes, numerals, letters, or other symbols. The complexity of patterns can also vary from simple alternating color or shape patterns to more complicated ones that involve mathematical operations (Collins & Laski, 2015) or multiple attributes. First-grade students were able to recognize patterns that consisted of numbers, letters, and other various objects with the same accuracy as significantly easier items such as colors or shapes. Researchers have determined that students are able to apply the same rule to patterns, even when the patterns are presented in different dimensions (Gadzichowski, 2012).

Instructional Strategies

The presentation of patterns does have an impact on learning. Educators need to be aware of the instructional strategies which also bolster patterning skills in order to maximize the benefits which patterning skills afford. Burgoyne (2019) concluded that alphanumeric patterns

were a higher predictor of achievement in reading and mathematics than non-alphanumeric patterns. Therefore, educators need to consider using letters and numerals when teaching patterns. Children who are just beginning to learn to pattern will find it easier to ascertain a pattern consisting of numerals if it is presented vertically, whereas letter patterns are easier to decipher when presented horizontally (Gadzichowski et al., 2014). In addition, it is important to provide examples and nonexamples of patterns. Challenging students to describe images that are not predictable and have no specific pattern allows students to become more aware of patterns when they encounter one (McGarvey, 2013). Tasmir et al. (2017) found that when students are asked to differentiate and compare a pattern and a non-pattern, they are able to differentiate which attributes are important and part of a pattern. Likewise, comparing two different patterns leads to higher reasoning skills. Children need to be given opportunities to notice, analyze, and describe images that have a predictable pattern and others that do not. Finally, students need to understand the relationship between the components of a pattern and have the opportunity to analyze the structure (Kidd et al., 2013).

In early childhood classrooms, explicit instruction can occur when students utilize manipulatives in a purposeful manner such as stringing beads in a repeating pattern. Patterns can also be taught indirectly by creating mathematical practice that incorporates predictable numerical sequences embedded within other mathematical learning (Fyfe et al., 2017). Explicit high-quality instruction is imperative in early math so that students, especially lower socio-economic students, develop a strong background in mathematics (Clements & Sarama, 2011).

Loehr et al. (2014) also determined that explicitly teaching patterning skills and/or allowing students to explore patterns on their own proved to be an effective manner in which to improve patterning knowledge. Students are able to solve complex patterning prior to

kindergarten as long as the teacher is able to incorporate the concept of unit of repeat into their pedagogy (Papic et al., 2011). When students learn to break a pattern down into equal-sized groups, it is more likely the student will utilize counting skills, thus connecting numeracy to patterning. Likewise, if students use only a pattern matching strategy when duplicating or matching a pattern, research has shown that they may only see the pattern as individual components rather than realizing that a rule exists within the pattern that guides the repeating unit (Collins & Laski, 2015).

The most important part of teaching patterning is not making complex patterns, but rather identifying the repeating part and connecting that information to other aspects of mathematics (Warren et al., 2012). Asking a child to identify what comes next in a sequence of items takes the focus off the repeating unit and puts it on one individual component of the pattern (McGarvey, 2013) because extending a pattern may not lead to the recognition of its structure (Tsamir et al., 2017). However, changing one attribute within the pattern may lead to students being able to see the repeating unit of the pattern (Warren, et al., 2012). Collins and Laski (2015) expanded on this concept when their research demonstrated the likelihood that relational reasoning is enhanced when patterning activities require students to reconstruct the repeating part of a pattern. Emphasizing how repeating patterns are structured can assist children in with both verbal and number systems (Zippert et al., 2020). However, the recognition of the structure of the pattern does not necessarily develop while extending patterns (Papic, Mitchelmore & Mulligan, 2011).

Growing patterns are best taught when students learn to visual the structure of the pattern. When teachers intentionally direct a student's attention to visualization, it is easier for students to connect the pattern’s components. One method of achieving this is to have students color in different parts of the growing pattern so that students can see the growing pattern. In addition,

students should be encouraged to visualize the pattern in more than one way. Educators need to be cognizant of the design of growing patterns and use this knowledge to intentionally inform instruction (Wilkie & Clarke, 2016).

A recent study by Pasnak et al. (2019) solidified that explicit instruction did improve the patterning abilities of kindergarten students. Students were given a daily 15-minute intervention in either patterning, literacy, or mathematics over the course of the school year. The students who received the patterning intervention scored 36% higher on patterning when compared to the group which received the literacy intervention and 27% higher than those in the mathematics intervention group.

In terms of identifying the repeating part of a pattern, the terms used when presenting a pattern are important since labeling a pattern appears to be a fundamental part of teaching patterning strategies. Students need to learn to describe the components of the repetition and how they relate to each other in terms of structure using mathematical language. Teachers need to encourage students to look for mathematical properties within patterns such as orientation, shape, and size and use this information to describe and predict how the pattern repeats mathematically (McGarvey, 2013).

Later research has not disagreed with the concept of describing patterns using mathematically terms, but it has also shown that patterning skills benefited more from the use of abstract labels such as ABAB instead of concrete labels (yellow, green, yellow, green) for three reasons. First of all, labels allow students to link the concrete materials to the abstract concept of the given pattern. Secondly, labels such as ABAB, can be used with a variety of materials and different types of patterns, thus giving students a method of verbalizing patterns consistently.

Lastly, looking at the concrete pattern while labeling it abstractly allows students to see the pattern and hear the abstract labels thus solidifying the two (Fyfe et al., 2015).

Building on the idea of the importance of abstract labeling, a study was conducted whereas the repeating part of a pattern was labeled using three different abstract labels and a fourth pattern that did not use any labels. The three abstract labels were letters (ABB), numbers (122) and groupings (one two). The researchers discovered that labeling patterns with abstract labels were more beneficial to a child's learning than not using any label. In addition, the study found that grouping labels may bring more attention to the number of objects in a pattern and thus make students more attuned to how many objects make up the pattern. The researchers concluded that "these results suggest that the way patterns are talked about can change the way patterns are thought about" (Flynn et al., 2020, p. 148).

The simplest method for increasing pattern knowledge with regards to early learners is including patterning in classroom activities; talking about patterns and doing pattern activities has a positive effect on pattern knowledge (Rittle-Johnson et al., 2015). Many early childhood educators believe that by allowing students to do puzzles or build with blocks, they are teaching mathematics. Whereas these skills do contain some mathematical content, they tend to focus more on fine motor skills or reading than on mathematics. This approach to mathematical learning is considered insufficient because it does not explicitly teach concepts, such as patterning, which are an integral part of mathematical learning (Clements & Sarama, 2011).

Early learning educators should teach patterning skills both directly and indirectly. Teachers must incorporate the teaching of patterns into their curriculum. Intentionally incorporating patterning skills within the daily routine of early childhood classrooms promotes an awareness and understanding of a variety of patterning skills. Utilizing one's body to create a

pattern such as head-shoulders-knees (Reid & Andrews, 2016), sitting boy-girl-boy-girl, incorporating patterns into the classroom calendar numbers, or searching for patterns in their environment are all productive methods for increasing pattern awareness (Wang, 2016). Classrooms provide an environment that helps promote curiosity and learning about math and need to be full of rich materials that promote math education (McClellan, 2014).

The National Council of Teachers of Mathematics (NCTM) also considers manipulatives to be an important component of math education. NCTM states that using manipulatives to distinguish patterns allows students to build a foundation in mathematical concepts. Incorporating patterns into all aspects of the curriculum is important, and there are many strategies that can enhance the learning of patterning (National Council of Teachers of Mathematics, n.d.).

Literacy is a mainstay of early childhood classrooms and should be integrated with mathematics in regard to patterning skills. The first way to incorporate math into literacy is by providing students with literature that encompasses authentic patterning within the story. This inclusion of literature allows for relationships between objects to be discussed. Teachers must pose questions that elicit mathematical responses such as "what pattern do you see?" and then have students create a drawing to represent the pattern (Whitin & Whitin, 2011). Wang (2016) noted three pieces of literature that encompassed patterning as part of the story. These are *ABABA a Book of Pattern Play* written by Brian Harris and two stories by Trudy Harris: *Pattern Fish* and *Pattern Bug*. Counting books also aid in the development of patterning skills. Books such as *One is a Snail, Ten is a Crab,* highlight skills such as the patterns of even and odd numbers (Whitin & Whitin, 2011).

One instructional strategy that is often overlooked when students are learning to decipher patterns is the role music plays. Music is often a child's first patterning experience since it engages even the youngest of children. Rhythmic beats and repeating words all add patterning experiences to young children's mathematical repertoire. Researchers investigated the role of music and patterns and found that students who did not have music integrated into mathematics had more difficulty recalling items that were taught in a math lesson (Geist et al., 2012).

Zippert et al. (2016) researched the correlation between a child's ability to find patterns in sound and compared it to visual patterning skills. Overall, the three- and four-year-old students did better on the visual tasks than on the auditory ones. However, both the auditory and visual measures proved to be good indicators of a student's pattern knowledge especially with regards to copying a pattern. The researchers concluded that music did play a role in patterning, but more research is needed to ascertain how music impacts a child's knowledge of patterns and vice versa.

Research has shown that patterns are an essential component of early education. Teachers need to be cognizant of their pedagogical practices in order to facilitate strategies that promote the learning of patterning skills. Students need the opportunity to learn patterning skills with a variety of patterns that are presented in varying ways, and adults need to facilitate instruction that leads students to transfer their knowledge of patterns to different aspects of mathematics (Papic, Mulligan & Mitchelmore, 2011).

Conclusion

The importance of mathematics in early childhood education has come to the forefront in recent years. Researchers have determined that a sound foundation in early mathematical skills

is consistent with increased achievement in mathematics in later grades (Clements & Sarama, 2011). Patterns and patterning are key components of early mathematics and provide students the opportunity to develop valuable mathematical reasoning skills. Children learn to think logically and gain valuable problem-solving skills when learning to analyze patterns (Wang, 2016). Recent studies have recognized the importance of patterning skills and have solidified the importance of patterning as an essential element of math curriculums (Burgoyne et al., 2019; Fyfe et al., 2016; Kidd et al., 2013; Lee et al., 2011; Nguyen et al., 2016, Rittle-Johnson et al., 2017; Rittle-Johnson et al., 2019; & Zippert et al., 2020).

This literature review has shown that incorporating patterning into the mathematics curriculum of early learners has a positive impact not only on their ability to understand and analyze patterns but also in its effect on future mathematics achievement. Patterning increases knowledge of numeracy and calculation (Fyfe et al., 2017, Zippert et al., 2020). Students who are proficient with patterns show increased abilities with regards to counting and solving equations as well as algebraic reasoning.

Algebra requires students to look not only for similarities and differences but also for the relationships between the parts of the whole. Students must ascertain and define the rule(s) that overarch the problem and utilize symbols to represent it (McGarvey, 2013). Likewise, patterning skills are built on the same premise. Learners must not only visual and analyze the similarities and differences that are essential to a given pattern, but most importantly they must learn to find the rule which governs the pattern (Papic et al., 2011, Warren et al., 2012, McGarvey, 2013, Collins & Laski, 2015). Papic, Mulligan and Mitchelmore (2011) believed that children as young as four were capable of developing advanced strategies for solving patterns and could actually be able to do pre-algebraic thinking.

Ironically, the Common Core does not currently include patterning skills as a standard for early learners. Sparse evidence on patterning existed when the standards were written in 2010. Research at that time was able to demonstrate a correlation between patterning and mathematics, but the evidence did not result in patterning being listed as a standard in early math education. Current research clearly indicates the benefits of analyzing and understanding patterns. Many researchers hope that patterning will become a standard within the Common Core for grades kindergarten through second in order to demonstrate the importance of patterning in early math education (Rittle-Johnson et al., 2019). Adding patterning as a standard in the Common Core would cement the significance of patterning and its effects on mathematical achievement (Rittle-Johnson et al., 2018). All stakeholders must work together to raise awareness of the importance of early math education (National Association for the Education of Young Children, 2010).

Research has determined that patterning needs to be explicitly taught in early childhood classrooms. Specific instructional strategies enable educators to actively encourage students to analyze and understand patterns. Educators, especially those of early learners, need to be cognizant of the importance of patterning for future mathematics achievement and incorporate patterning opportunities into the curriculum that elicit later mathematical learning (Collins & Laski, 2015). Students who receive explicit instruction on patterning grow exponentially not only in patterning and problem-solving skills, but also with algebraic reasoning as well. Teachers should incorporate specific teaching strategies to address patterning skills into their pedagogy to increase knowledge and awareness of patterns not only in the classroom but also in the world around them.

Since children begin to develop math skills at a very young age, it is crucial that research studies continue to address the skills and strategies that result in the highest level of

mathematical achievement. Overall, the question of how students understand patterns and how this understanding results in academic performance is key to future research (Kidd et al., 2013), and future research should be based on evidence-based practices (Rittle-Johnson et al., 2019). Young learners are capable of working with complex and advanced patterns (Papic, Mulligan & Mitchelmore, 2011), and researching the best way to cultivate this knowledge is key. Researchers also suggest professional development for educators to help them comprehend the capabilities of early learners in regard to mathematics (Mulligan et al., 2020). Furthermore, studies need to use current data to begin to ascertain the trajectory of learning for students to encompass the importance of patterning at an early age (Inchaustegui & Alsina, 2020).

Research has shown that such patterning instruction improves both reading and mathematics, so it is necessary that studies continue to focus on the importance of patterning as a valuable component of mathematical instruction (Kidd et al., 2013). Schmerold et al. (2016) notes that older students might also benefit from instruction in patterning.

It is crucial that early learners be exposed to mathematical skills and concepts that have been proven to result in higher levels of mathematical achievement. Patterning accomplishes that task. Research-based studies have shown empirical evidence that teaching and learning patterning has a direct result on future mathematical achievement. Therefore, teaching and learning patterning is a critical and necessary component of mathematical instruction and needs to be incorporated into the curriculums of early childhood classrooms.

References

Bock, A. M., Cartwright, K. B., McKnight, P. E., Patterson, A. B., Shriver, A. G., Leaf, B. M., Mohtasham, M. K., Vennergrund, K. C., & Pasnak, R. (2018). Patterning, reading, and executive functions. *Frontiers in Psychology, 9,* 1802. https://doi.org/10.3389 fpsyg.2018.01802

Burgoyne, K., Witteveen, K., Tolan, A., Malone, S., & Hulme, C. (2017). Pattern understanding: Relationships with arithmetic and reading development. *Child Development Perspectives*, *11*(4), 239–244. https://doi.org/10.1111/cdep.12240

Burton, G. M. (1982). Patterning: Powerful play. *School Science and Mathematics, 82*(1), 39–44.

Clements, D., & Samara, J. (2011). Early childhood mathematics intervention. *Science 333*, 968-970. https://doi:10.1126/science.1204537

Collins, M. & Laski, E. (2015). Preschoolers' strategies for solving visual pattern tasks. *Early Childhood Research Quarterly 32*, 204-214.

Flynn, M. E., Guba, T. P., & Fyfe, E. R. (2020). ABBABB or 1212: abstract language facilitates children's early patterning skills. *Journal of Experimental Child Psychology*, *193*, 1-15.

Fyfe, E. R., McNeil, N. M., & Rittle-Johnson, B. (2015). Easy as ABCABC: abstract language facilitates performance on a concrete patterning task. *Child Development*, *86*(3), 927 935. https://doi.org/10.1111/cdev.12331

Fyfe, E. R., Evans, J.L., Matz, L.M., Hunt, K. M., & Alibali, M. W. (2017). Relation between patterning skill and differing aspects of early mathematical knowledge. *Cognitive Development 44*, 1-11.

Gadzichowski, K. M. (2012). Patterning abilities of first grade children: Effects of dimension and type. *Creative Education 3*(5), 632-635.

Gadzichowski, K. M., O'Brien, S. E., & Pasnak, R. (2014). Orientation of letter and number patterns. *Journal of Education and Human Development 3*(2), 59-66.

Geist, K., Geist, E., & Kuznik, K. (2012). The patterns of music: Young children learning mathematics through beat, rhythm, and melody. *Young Children (67),* 74-79.

Hutchinson, E. & Pournara, C. (2011). Pre-school children's understanding of mathematical patterns. *South African Journal of Childhood Education*, *1*(2), 92–111.

Inchaustegui, Y. & Alsina, A. (2020). Learning patterns at three years old: Contributions of a learning trajectory and teaching itinerary. *Australasian Journal of Early Childhood 45*(1).

Iowa Department of Education (2018). *Iowa early learning standards*. Retrieved September 16, 2020 from https://educateiowa.gov/sites/files/ed/documentsIowaEarlyLearningStandards-3rdEdition12.20.18_508.pdf

Johnson, D. (1968). Enjoy the mathematics you teach. *The Arithmetic Teacher, 15*(4), 328-332. Retrieved October 4, 2020, from http://www.jstor.org/stable/41185769

Kandir, A., Colak, F. G., Aktulun, O. U. (2018, November 23). The effect of pattern-based mathematics education program (PMEP) on 61-72-month-old preschoolers' early academic and language skills. *Educational Research and Reviews 13*(22), 735-744.

Kidd, J. K., Carlson, A. G., Gadzichowski, K. M., Boyer, C. E., Gallington, D. A., & Pasnak, R. (2013). Effects of patterning instruction on the academic achievement of 1st-grade children. *Journal of Research in Childhood Education*, *27*(2), 224-228. https:/link.gale.com/apps/doc/A329301783/AONEu=sliowa_main&sid=AONE&xid=ff533123

Lee, K., Ng, S. F., Bull, R., Pe, M. L., & Ho, R. H. M. (2011). Are patterns important? An investigation of the relationships between proficiencies in patterns, computation, executive functioning, and algebraic word problems. *Journal of Educational Psychology,103*(2), 269–281. https://doi.org/10.1037/a0023068

Loehr, A., Fyfe, E., Miller, M., & Rittle-Johnson, B. (2014, April). Learning from explanations: Does it matter who provides them? Paper presented at the annual meeting of the American Educational Research Association (AERA), Philadelphia, PA.

Lüken, M., Peter-Koop, A., & Kollhoff, S. (2014). Influence of early repeating patterning ability on school mathematics learning. In P. Liljedahl, C. Nicol, S. Oesterle, & D. Allan (Eds.), *Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education. Vol. 4* (pp. 137-144). Vancouver, Canada: PME.

McClellan, D. (2014). Making math meaningful for young children. *Teaching Young Children 8*(1). Retrieved from https://www.naeyc.org/resources/ pubs/tyc/oct2014 making-math-meaningful

McGarvey, L. M. (2013). Is it a pattern? Teaching and learning patterns in early childhood mathematics. *Teaching Children Mathematics, 19*(9), 564-571.

Mulligan, J., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal*, *21*(2), 33–49.

Mulligan, J., Oslington, G. & English, L. (2020) Supporting early mathematical development through a "pattern and structure" intervention program. *ZDM Mathematics Education* 52, 663–676. https://doi.org/10.1007/s11858-020-01147-9

National Association for the Education of Young Children (2010). Early childhood

mathematics: Promoting good beginnings. Retrieved September 24, 2020 from naeyc.org

National Council of Teachers of Mathematics (n.d.). Algebra as a strand of teaching mathematics to all students: A position of the national council of teachers of mathematics. Retrieved September 16, 2020 from https://www.nctm.org/Standards-and-Positions/Position-Statements/Algebra-as-a- Strand- of -School-Mathematics-for-All-Students

National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common core state standards mathematics*. National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington D.C. Retrieved September 10, 2020 from http://www.corestandards.org

Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D. H., Sarama, J. S., Wolfe, C., & Spitler, M.E. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement?. *Early Childhood Research Quarterly, 36*, 550–560. https://doi.org/10.1016/j.ecresq.2016.02.003

Papic, M. & Mulligan, J. (2007). *The growth of early mathematical patterning: An intervention study***.** 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart, Vol.2, p. 591-600 Retrieved September 30, 2020 from https://eric.ed.gov/?id=ED503746

Papic, M., Mulligan, J., & Mitchelmore, M. (2011). Assessing the development of preschoolers' mathematical patterning. *Journal for Research in Mathematics Education*, *42*(3), 237 269.

Pasnak, R. (2017). Empirical studies of patterning. *Grantee Submission, 8*, 2276-2293. Retrieved from http://ezproxy.nwciowa.edu/login?url=https://www-proquest-com.ezproxy. nwciowa. edu/docview/2034281333?accountid=28306

Pasnak, R., Gagliano, K., Righi, M., and Kidd, J. (2019). Teaching early literacy, mathematics, and patterning to kindergartners. *Psychology, 10*, 1493-1505. doi: 10.4236/psych. 2019. 1011098.

Plessis, J. (2018). Early algebra: Repeating pattern and structural thinking at foundation phase. *South African Journal of Early Childhood Education, 8* (2). http://dx.doi.org/ 10.4102/ sajce.v8i2.578

Reid, K., & Andrews, N. (2016). Fostering understanding of early Numeracy development. Australian Council for Educational Research (ACER). https://research.acer.edu .au/ monitoring learning/29

Rittle-Johnson, B., Fyfe, E. R., Hofer, K. G., & Farran, D. C. (2017). Early math trajectories: low-income children's mathematics knowledge from ages 4 to 11. *Child Development*, *88*(5), 1727–1742. https://doi.org/10.1111/cdev.12662

Rittle-Johnson, B., Fyfe, E. R., Loehr, A. M., & Miller, M. R. (2015). Beyond numeracy in preschool: adding patterns to the equation. *Early Childhood Research Quarterly*, *31*, 101–112. https://doi.org/10.1016/j.ecresq.2015.01.005

Rittle-Johnson, B., Zippert, E.L., Boice, K. L. (2019). The roles of patterning and spatial skills in early mathematics development. *Early Childhood Research Quarterly*. DOI: 10.1016 j.ecresq.2018.03.0

Schmerold, K., Bock, A., Peterson, M., Leaf, B., Vennergrund, K., & Pasnak, R. (2016, November). The relations between patterning, executive function, and mathematics.

The Journal of Psychology Interdisciplinary and Applied. http://dx.doi.org 10.1080/00223980.2016.1252708

Steen, L. (1988, April 29). The science of patterns. *Science, 240*(4852) 611-616.

Tsamir, P., Tirosh, D., Levenson, E., Barkai, R., & Tabach, M. (2017). Repeating patterns in kindergarten: Findings from children's enactments of two activities. *Educational Studies in Mathematics 96* (1).

Thorton, C. (1977). A glance at the power of patterns. *The Arithmetic Teacher, 24*(2), 154-157. Retrieved October 10, 2020, from http://www.jstor.org/stable/41191349

Wang, Q. (2016). Mathematical patterning activities in the early grades. *Ohio Journal of School Mathematics, 73,* 37-42.

Warren, E., Miller, J., & Cooper, T. (2012). Repeating patterns: strategies to assist young Students to generalize the mathematical structure. Australasian Journal of Early Childhood, 37(3), 111+. doc/A330997675/AONEu=nwcollege &sid=ONE& xid= e857afc7

Wilkie, K. & Clarke, D. (2016). Developing students' functional thinking in algebra through different visualizations of a growing pattern's structure. *Mathematical Educational Research Journal 28*, 223-243.

Whitin, P., & Whitin, D. J. (2011). Mathematical pattern hunters. *Young Children, 66*(3), 84–90.

Zippert, E., Gross, K., & Rittle-Johnson, B. (2016). Measuring preschoolers' sound pattern relations to spatial and numerical understanding.10.13140/ RG.2.2.28838.16965. https://peabody.vanderbilt.edu/departments/psych/research/ research_labs/childrens_learning.lab/ MMSPoster.pdf

Zippert, E. L., Douglas, A.-A., & Rittle-Johnson, B. (2020). Finding patterns in objects and numbers: repeating patterning in pre-k predicts kindergarten mathematics knowledge. *Journal of Experimental Child Psychology, 2*00. https://doi.org/10.1016 j.jecp.2020.104965