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Number Sense and the Effects on Students' Mathematical Success

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Abstract

This literature review's main focus discussed how number sense effect students' mathematical achievement and what strategies teachers can use to build students' number sense. While researching number sense, it was often found that explicit instruction, teacher knowledge, and socio-economic status impacts students' knowledge of number sense. It has been found that the earlier students start learning simple math skills the better it will be on their math development as they go into kindergarten and get older. In knowing the importance of number sense, teachers have to know what strategies they should use to support all students build number sense. A common instructional strategy used to teach number sense, is explicit instruction. A few of the many instructional strategies explained in more detail include: Five frames, number lines, think alouds, and concrete to abstract representations. Teachers need to have the knowledge of how strategies can be used to support students' mathematical understanding. This literature review will also look at instructional strategies, future research, memorizing versus understanding, and the literacy connection of number sense.

Keywords: Number sense, explicit instruction, math achievement, early instruction, intervention, instructional strategies, concrete representation

Number Sense and the Effects on Students' Mathematical Success

Number sense is defined in a variety of different ways. To understand number sense, it can be broken up into several components. According to Berch (2005), number sense involves counting, number knowledge, number transformation, estimation, and Bryant, Bryant, Gersten, Scammacca, and Chavez (2008) include number patterns as an additional component of number sense to the other four areas listed. According to Gurganus (2004), when students have strong number sense, they know that numbers can be used and represented in a variety of ways. Specifically, students would know that numbers stand for an amount of something, a number is connected to other numbers, and numbers can be used to solve problems. Sood and Jitendra (2007) referenced specific types of number relationships that students need to develop. The first one is called spatial relationships. This is when you can look at an amount of something and identify or estimate the quantity quickly, this is often referred to as subitizing. An easy example of this would be identifying the amount on a number die. The end goal is for students to identify groups of objects that are more difficult than die formation. The second number relationship is knowing what comes one and two more and less than a specific number. The third number relationship is knowing number relationships of 5 and 10. This includes knowing two numbers that can be added or taken away fluently to reach 5 and 10. The fourth number relationship is part-part-whole. This is understanding that two numbers (two parts) make another number (whole) and the whole can be broken up into two parts (Van de Walle, 2007). Throughout this literature review, number sense is referred to as students understanding what numbers mean, how they relate to one another, and applying their number knowledge to other mathematical areas and application problems. Number sense is to math like phonemic awareness is to reading. When students have strong number sense, it helps them be successful in all mathematical areas. This literature review's main focus will discuss how number sense effect students' mathematical achievement and what strategies teachers can use to build students' number sense.

Literature Review

The research that is referenced in this literature review goes into detail about how early instruction at a young age impacts a child's mathematical understanding in their future. While researching number sense, it was often found that explicit instruction, teacher knowledge, and socio-economic status impacts students' knowledge of number sense. This literature review will examine those impacts, in addition to: History, instructional strategies, future research, memorizing versus understanding, and the literacy connection of number sense.

History

Nataraj and Thomas (2009) state that understanding the history of mathematics may help teachers come up with strategies to support their students' own understanding of mathematics. These researchers looked at the history of the Mayan and Indian mathematics. Nataraj and Thomas (2009) found that large numbers were used very often in the past, which allowed people to get comfortable with using big numbers in their daily lives. Powers of ten were used quite frequently before the base-ten system was used. Powers of ten required people to know how to multiply numbers together. Nataraj and Thomas (2009) gave detail of the development of the number system. Numbers were developed and used in the following order: Numbers verbally stated in a combination of expanded and word form (an example includes: five sata + six dasa + eight eka), numbers identified through symbols, a place value system was developed (using different words other than hundreds, tens, and ones), and finally the zero was incorporated in the number system. Nataraj and Thomas (2009) conducted a study of teacher instruction based on the historical development of numbers for 13-year olds. The study also looked at the effects of using concrete models to help understand numbers and whether instruction should begin with bigger numbers just like in the order numbers were developed. The study found that concrete models used in different forms of the different stages of the historical development to be beneficial to students' mathematical understanding. Teaching students with larger numbers also showed a positive effect. For some students it challenged them, but for others it was difficult which caused some students to give up. Therefore, it was mentioned that the students who really struggle, may need support and explicitly taught when dealing with larger numbers.

When looking at the history from an instructional view, a variety of methods and ideas were used similar to how they are today. Kamii and DeClark (1985) share that children should come up with their own strategies when learning about ones and tens from a development standpoint. Instead of using objects, Kamii and DeClark (1985) state that students should use mental math when adding and subtracting simple problems. This is similar to Courtney-Clarke and Wessels (2014) and Woods, Ketterlin Geller, and Basaraba, (2018) explaining that number sense should be directed through students' strategies and less focus on teacher direction. These researchers explained students should be challenged with higher leveled thinking and asked to share their understanding. Fuson (1990) states that concrete objects should be used to help students solve and understand addition and subtraction problems. In similarity, Woods et al. (2018) said the research shows that using concrete representations of numbers with explicit instruction will benefit students build their number sense. Sood and Jitendra (2007) informs that making instruction connect to the real world is beneficial to students' mathematical understanding. Bednarz and Janvier (1988) states an explicit model will help support numeracy learning for students struggling and looks at concepts rather than the algorithm. Jones, Thornton, and Putt (1994) created their own study based on the beliefs of the researchers listed above. Their study was created to determine what strategies to use with students to help build number sense using multidigit numbers. In their study, Jones, et al. (1994) wanted to build students' number sense so they made sure to incorporate the following into their framework: Counting, putting things in groups, putting things together and taking apart, and making connections between numbers. When looking at the components of researcher's beliefs from 1985 to 1994, the majority are similar ideas and beliefs that are still used today.

Early Instruction Impact

Infants can begin to develop number sense through play and discussion. They may begin to refer to numbers as how many they have of something or comparing amounts (Woods et al., 2018). Morgan, Farkas, and Wu (2009) state that students coming into kindergarten with little to no mathematical knowledge will show growth in their learning but at a slower pace than a student who already comes into kindergarten with mathematical understanding. When students come to school and do not have those basic number skills, that is when teachers want to provide direct and explicit instruction of what numbers mean and number relationships to prevent later mathematical difficulties. When a student has not grasped number sense, especially place value, that is a predictor the student will struggle in later grades. Bryant et al. (2008) says that students who struggle with computation fluency is due to the lack of basic number understanding. Therefore, students who struggle with computation, often will have a hard time with story problems. Number sense is the base of mathematics, and without it, students will eventually struggle.

Several different studies done with kindergarten students showed number sense strength to be a predictor of math success in those students' future. Jordan, Glutting, Dyson, Hassinger-Das and Irwin (2012) predicted math success in third grade when students were strong in mathematics in kindergarten. In addition to the same findings as Jordan et al. (2012), Ivrendi (2016) also found mathematical understanding in kindergarten to be a good predictor of overall academic areas in reading and science to 8th grade. Morgan et al. (2009) did a study that showed when students scored below the tenth percentile on a math assessment in kindergarten, they had a seventy percent change of still being below the tenth percentile five years later. Morgan et al. (2009) specifically looked at how students scored on math assessments in the fall and spring of their kindergarten school year. When students scored low on the fall and spring assessment, they had the slowest growth rate in their mathematical development. When students scored low on the spring assessment, they had a slow growth rate. When students scored low on only the fall assessment, their growth rate was not as affected as the other two groups of students. Students who were not low on either assessment showed the fastest growth rate among all the groups. Aunola et al. (2004) did a similar study to Jordan et al. (2012), Ivrendi (2016), and Morgan et al. (2009) but their study focused on students coming to preschool and watched students' math progression through second grade. This study looked at whether metacognitive knowledge, listening comprehension, and counting ability relates to a students' mathematical ability. It was found that students' knowledge and ability of counting was the strongest predictor of students' math skills as they progressed in their schooling. Counting in this study included students counting verbally as high as they could go, counting on from a given number, counting backward from a given number, and being asked a quantity plus a given number. For example, the study asked students how many more is 8 if you start at 3, and students would know the answer to be 11. Aunola et al. (2004) also found that students that came into preschool with mathematical knowledge, especially counting knowledge, learned math content more quickly than students who came into preschool with less counting knowledge. When teachers intervene early on, it is shown to help students' mathematical success. Math intervention can take place as early as preschool. According to Notari-Syverson and Sadler (2008), an intervention study done with preschool students showed mathematical improvement in comparison to students who did not receive math intervention. The intervention group used Big Math for Little Kids Curriculum that involved songs, games, books, and students conversing with one another (Notari-Syverson & Sadler, 2008).

In summary of these studies, number sense impacts all parts of mathematics and without a strong understanding, students will struggle in all areas of math. The earlier students start learning simple math skills, the better it will be on their math development as they go into kindergarten and get older. In knowing the importance of number sense, teachers have to know what strategies they should use to support all students build number sense. A common instructional strategy used to teach number sense, is explicit instruction.

Explicit Instruction

While researching about number sense, the importance of using explicit instruction to teach number sense came up quite often. Explicit instruction has a strong impact on all students, especially students who struggle with mathematics. Woods et al. (2018) says the research shows that using concrete representations of numbers with explicit instruction will benefit students build their number sense. Kilpatrick, Jeremy, Swafford, Jane, and National Research Council (2002) explain that using concrete objects is to help students see the meaning behind the math skill to grasp a better understanding. The objects should not be used to solve problems for very long periods of time. When students can easily solve a problem using concrete objects and when they understand the skill using concrete representations, then it is time to move to a visual representation. Eventually the goal is to have students only need an abstract representation and understand the meaning behind the skill they are doing. For students to get to abstract understanding, it is found helpful for them to be provided with explicit instruction using concrete and visuals (Gersten et al., 2009). An example of concrete representation is using manipulatives, such as base-10 blocks. An example of a mathematical visual is a ten frame, which is often used to support understanding numbers and problem solving. An example of abstract representation would be for the student to not need the concrete or visual. Abstract thinking is when a person can solve a problem in his or her head by thinking about a strategy or having the answer automatically.

To clarify what explicit instruction looks like, it consists of modeling and thinking aloud how to understand or do a specific skill. Doabler and Fien (2013) add that modeling is using language and demonstrations that are easy for students to understand. After modeling, scaffolding needs to take place. Sood and Jitendra (2007) describe scaffolding as allowing the student to gradually take more ownership with teacher support and eventually taking away the teacher support. Sometimes the scaffolding does not focus on the teacher support and is more about the material support. An example may be to use a number line and the scaffold might begin with teacher guiding the student on using the number line, then working towards the student using the number line on his/her own, and eventually the goal would be to for the student to solve a problem without a number line. Doabler and Fien (2013) agrees that scaffolding is needed after modeling to allow students to practice for independency. In addition, Doabler and Fien (2013) add that pre-teaching and review of the content is part of the guided practice stage. When providing explicit instruction, instant and specific feedback needs to be provided (Sood & Jitendra, 2007). Doabler and Fien (2013) agree that feedback should be instant, continuous, and errors corrected in a positive manner. Instant means that the feedback is told to students as soon as possible. Preferably right when a student makes an error so that the student does not think he or she is correct and develops an incorrect understanding or habit. Specific feedback means that the teacher reinstates what the student did correctly or incorrectly. An example of specific feedback might be, "You did a nice job showing that five can be broken up with four and one using the ten frame" or "That is incorrect, you got mixed up by going too fast, let's go back and point and count to each object one at a time." A non-example would be saying, "Good job" or "That is incorrect." The last piece of explicit instruction that Sood and Jitendra (2007) considers, is continual and long-term review of a skill so that it stays in the students' long-term memory. This means that review of a skill is consistent and independent practice is done over a period of time.

While researching explicit instruction, studies done in comparing different number naming systems were done by Magargee and Beauford (2016). Explicit number names are used in the Mandarin language. Magargee and Beauford (2016) did a study with pre-kindergarten and kindergarten students in Texas where students were taught using explicit number names and then gradually was introduced to the English number name systems. Explicit number name system focuses on the place value. An example of saying a number using an explicit number name would be: Two-ten six for the number 26. The study showed students benefit from being taught using explicit number names and traditional number names. Using explicit number names can create a stronger number sense due to the fact that the name is more focused on the place value (Magargee & Beauford, 2016). Gersten and Chard (1999) state that for number sense instruction to be effective, explicit instruction should occur instead of discovery or implicit learning. Latterell (2003) mention that students should explore by using what they have learned with new experiences to help them build problem solving skills.

Star (2016) looks at the upcoming curriculum and pedagogical changes that are wanting to be made in math instruction. Star (2016) refers to the discussion of student led versus teacher led instruction. New standards are wanting students to be involved in discussion and learning to apply their knowledge to real world situations. According to Star (2016), teacher-led instruction is the most common method used by teachers. Star (2016) clarifies that instead of teachers changing so many things at once, that teachers should revamp the way they conduct teacher-led instruction just by a few different things at a time. One change in teacher-led instruction Star (2016) mentions teachers could make is to use questions that get students to use higher-level thinking. An example of higher-level thinking would be asking questions that requires a more than one-word response. The second change Star (2016) states could be done is to show worked examples to students. A worked example is showing students every step in solving a problem. It is also important that teachers choose problems wisely that connect with the lesson objective and can provide discussion among students. Star (2016) informs the final change teachers can make to improve teacher-led instruction is to show different ways of solving problems. This allows students to find a strategy that makes most sense to them. Explicit instruction is found to be very beneficial for all students. The main components of explicit instruction include modeling, guided to independent practice, instant and positive feedback. There are a variety of strategies teachers can use to build students' number sense while conducting explicit instruction.

Instructional Strategies

Clarke et al. (2011) states that a solid tier one math program has to focus on teaching the essential skills students need to know and teachers need to use research based instructional practices. It is hard to find programs that already exist with these two components. Clarke et al. (2011) created a program called Early Learning in Mathematics (ELM) for kindergarten students. There are 120 lessons that last 45 minutes long and calendar lessons that last 15 minutes long. After the fourth lesson, the program has problems that students try to solve as a whole class. The problems' focus will require students to use their skills in measurement, geometry, or numbers and operations. Vocabulary is also incorporated in the lessons as a focus area for students. ELM specifically teaches numbers through 100. Many kindergarteners will not be ready to work with numbers that high, but it will still be taught for those students who are ready. The main focus of ELM will be working with numbers 1-30 by counting, counting with one to one correspondence, taking numbers apart, adding one to a number, and solving story problems by using addition and subtraction strategies. When teaching geometry to kindergarten students, ELM's main focus is that students hear names of two and three-dimensional shapes and can identify them when given a shape. ELM also touches on teaching students about simple patterns, measuring objects to compare length, time and money. The majority of the lessons focus on numbers and operations, whereas measurement and geometry have less time devoted in that area. Clarke et al. (2011) made sure that ELM uses research based instructional practices. This involves the gradual release model, which includes teacher models and gradually allows students to practice the skill. ELM also includes showing the skill being taught in different forms. That means students see something physically, then it is shown with a picture, and the goal is for students to be able to complete the task without any representation. The third instructional strategy that ELM uses is having students do think alouds. A think aloud is when someone shares what he or she thought in his or her head. Teachers may have to model think alouds many times and guide students before students are able to do them on their own. The goal is for students to share out loud what they thought or how they solved a problem in their head so that students can learn from one another. The last instructional strategy ELM includes are ongoing assessments to check for understanding of current and past content. When ELM was used in the study conducted by Clarke et al. (2011), students at risk showed more achievement than students not at risk. This shows that ELM is especially beneficial for struggling students in hopes to close their achievement gap.

Andrews and Sayers (2015) conduced a research study using a framework they created called the Foundational Number Sense (FONS). The purpose of this study was to create a framework that included skills that help build number sense. Within the framework are specific examples of how teachers can help students practice each number sense skill. The eight skills include: Identifying numbers, counting on and counting backwards from a given number, comparing numbers to an amount of objects, recognizing differences among quantities, showing numbers in a variety of ways, estimating, patterns, and solving basic operations (Andrews & Sayers, 2015). For each skill, Andrews and Sayers (2015) provided an example of how students can practice the skill. The example of systematic counting involves the student counting on and back from a specific number. In their study, they had three different cultures teach 1st grade lessons that would reach as many of these skills as possible. They found the framework to be beneficial but would like to research further in how the different cultures learn the eight skills.

Bryant et al. (2008) did a study with first and second grade students in a tiered two intervention group. These students received an additional fifteen-minute instruction time for 18 weeks using a framework that they created on their own called "Booster" lessons. When looking at this framework in comparison to Andrews and Sayers (2015), there are a few similar skills that were focused on in both frameworks. Bryant et al. (2008) had number recognition and systematic counting. The additional skill that Bryant et al. (2008) had was writing numbers and skip counting. It was noted that first graders are expected to count, identify, and write numbers to 99 and second graders are expected to do those skills to 999. Comparing numbers is also a similar skill, but on the Bryant et al.'s (2008) framework, it provides the strategy of using a five frame, ten frame, and part-part-whole relationships. Both frameworks have student practice addition and subtraction. Bryant et al. (2008) provides more specific strategies such as: "Fact families, doubles, doubles +1, make 10, count down -1, -2, and number bonds" (p. 27). A difference in the Bryant et al. (2008) framework, is place value. In first grade, they had students working with 10s and 1s and by second grade they had students working with 100s, 10s, and 1s. A few other strategies that Bryant et al. (2008) used for place value was using the base-10 model. Base-10 would include: five 100s, one 10, eight 1s. Within this model, students would be required to read and write numbers by using the base-10 model and be able to state the place value each number is in. For example, for the number 518, a student would be able to say that there is five 100s. Bryant et al. (2008) also used place value by using the standard model. The standard model would be saying 518. Bryant et al. (2008) shares similar statements to Gersten and Chard (1999) in how instruction should be provided explicitly to students. The intervention time in the Bryant, et al. (2008) study was implemented through a similar form of the gradual release model. It began with the teacher modeling, the teacher thinking aloud to students, having the students do the task with assistance (guided practice), pacing, and the teacher gave feedback to students on how they did the task. A similarity to Sood and Jitendra's (2007) study and Bryant et al.'s (2008) study is the use of concrete to semi-concrete practice with students to develop a sense of understanding with mathematical ideas. According to Bryant et al. (2008), teachers used concrete objects as needed (such as base-ten blocks), then used visuals (such as a number line) and worked towards no visual or concrete tool to support scaffolding for students. At the end of the study, it was found that the tier two intervention was beneficial for the second-grade students but not for the first-grade students. The researchers state that this may be due to the fact that the first graders needed more time to develop all of the skills that was being asked of them, but that is an assumption until further research is done.

Witzel, Ferguson, and Mink (2012) share three strategies to help build number sense. In agreeance with other researchers, using concrete materials allows students to build an understanding with a use of materials. Witzel et al. (2012) points out that teachers have to pay attention to what each student needs to continue to grow their understanding. Some students will need to use concrete materials to solve math problems while other students are ready to move onto a picture model or even ready to do it in their head. The second strategy shared by Witzel et al. (2012) is to teach to proficiency. Teachers have to teach so many skills, but Witzel et al. (2012) reminds the importance of students understanding an essential skill before moving onto a skill more complex. The third strategy Witzel et al. (2012) listed was to teach and use math language, and connect it to other content areas or activities so that students see the connection math has in the real world. In conjunction with math language, Witzel et al. (2012) shares that student think alouds are important to building students' mathematical understanding just like Clarke et al. (2011) incorporated in the ELM program.

McGuire, Kinzie, and Berch (2012) provides five frames as an instructional strategy that can help improve students' knowledge of numbers 0 through 5. A five frame is a 1 X 5 row of squares that is usually used to put circles, or counters in, to represent a number 0 through 5. McGuire et al. (2012) references several researchers to explain why five frames can benefit students' mathematical development. Novakowski (2007) says that five frames give students a visual and connection to the number five so that when students are dealing with number combinations under five, some might refer to the five frame visual. Wynn (1990) points out that five frames are a small visual for students so that they are not overloaded with a bigger amount of something they are not ready for, such as a ten frame. The National Research Council (2009) informs that five frames can provide students with another representation for the numbers 0-5. Some students can easily connect that the empty squares on a five frame is the same as how many fingers they have on one hand. Hunting (2003) mentions that five frames can support the part-part-whole concept within five. For example, a teacher could show 3 red colored circles and 2 yellow colored circles to demonstrate that one part is 3, one part is 2, and the whole is 5. Finally, McGuire et al. (2012) considers five frames to prepare students for ten frames, which would be the next step when students are proficient with numbers 0 to 5.

Woods et al. (2018) shares that a number line can be used as an instructional strategy to construct number sense. When given instruction on how to use number lines, they can teach students a variety of skills such as: Counting, ordering of numbers before and after, patterns, and learning how far apart numbers are from one another. However, to allow number lines to be used effectively, students (especially students with learning difficulties) need to be given explicit instruction on how to use the number lines (Woods et al., 2018).

Small groups will be beneficial for students during math instruction. Small groups can also be helpful to the teacher to see what students understand and what reteaching may need to occur (Bryant et al., 2008). Gersten et al. (2007) agree that small groups of no more than six students can help students struggling with mathematics or any content area when they are given explicit 30-minute instructional periods. Kilpatrick et al. (2002) agree that small groups can be beneficial to student learning when they are used in the right way. That means students are taught their expectations of how to act and know what they will be doing in the group. Kilpatrick et al. (2002) makes a point that if not all students participate in the group, then it makes the group time less effective. Making instruction connect to the real world is beneficial to students' mathematical understanding. When provided explicit instruction and then make connections with the task on how students may use that information in the real world, will give students something to make a connection to and instill that learning in their long-term memory (Sood & Jitendra, 2007).

Two sources looked at the benefits of board games to support number sense. Woods et al. (2018) and Jordan et al. (2012) state that number paths on a board game can build a student's understanding of length. What was found to be more beneficial was when the board game had numbers on the board so that when you spun, you moved forward which allowed students to learn what came before and after numbers. Wiest (2006) explain that games in general can support students' number sense when the games are chosen with purpose and students play the game correctly to support the learning.

Kilpatrick et al. (2002) mention that instruction should try to include as many math skills as possible in a lesson. Kilpatrick et al. (2002) created these five math skills that will be referred to as math branches in this literature review. One branch is to understand the math ideas and know the underlying meaning of math concepts. When students understand the math skill, it allows them to work less at solving other math problems because they can find connections to what they've learned and know. According to Kilpatrick et al. (2002), when students have a good understanding of a mathematical concept, they should be more likely to catch when a problem is incorrect. The second branch is actually solving the math problems. Kilpatrick et al. (2002) consider fluency and accuracy to be important in this branch. For students to solve the problems accurately and fluently, they need to truly understand the problem. The third branch is being able to apply a math skill to a real-world problem. For students to apply what they know, they need to understand the mathematical problem and know what method to use or come up with a solution on their own. The fourth branch is a step beyond the understanding branch and the student is able to explain the process or product. The fifth branch is the will to do the work and knowing why and how each mathematical problem is important. Kilpatrick et al. (2002) state that the goal is for students to be math proficient and that means that these five branches are intermixed as much as possible when teaching students to develop their mathematical skills. Students are more likely to remember the mathematics when more than one branch is being incorporated as it will make the experience and learning more memorable to enlist in their long-term memory. In addition to these five math branches, Kilpatrick et al. (2002) also mentions the importance of a welcoming classroom where students feel comfortable sharing ideas with one another and not afraid to ask questions. The teacher should also make sure that students have enough independent practice time and that the practice should be longer than the guided and modeled practice. During this practice time, students may be working independently, with partners, or in small groups (Kilpatrick et al., 2002). The instructional strategies listed above can be used in conjunction with an explicit teaching model. Teachers need to have the knowledge of how those strategies can be used to support students' mathematical understanding.

Teachers' Content Knowledge

Kilpatrick et al. (2002) says that teachers need extra knowledge about math to effectively teach math. Not only do they need to know how to solve math problems, they also need to understand the concepts in a variety of ways to help students who think of different methods. According to Darling-Hammond, (2000) teachers' understanding of math and their knowledge on how to teach mathematics to students is the greatest variable of students' math achievement. Hill (2008) held a study that had similar findings in which the teachers' knowledge of math and how they teach the concepts, impacts students' understanding of mathematics. Courtney-Clarke and Wessels (2014) discuss that if our teachers do not have the knowledge of mathematics that is going to support our students, then we have to go back to teacher preparation programs and make sure pre-service teachers are learning what they need to help students in their classroom. Kilpatrick et al. (2002) agrees that colleges need to prepare teachers to think how students may understand problems and the stages students go through in math development.

Yang (2007) did a study where fifteen pre-service teachers in Taiwan were interviewed by solving number sense problems. The way the pre-service teachers solved the problems were analyzed by whether they used a number sense-based strategy, a rule-based strategy, or could not explain their solution correctly. It was found that about five teachers mainly used number sense strategies, and about ten of the teachers used rule-based strategies. It was noted that teachers had a hard time estimating when the problem asked the teachers to estimate. Instead, the teachers wanted to provide the exact answer. These results conclude with other research that number sense instruction needs to take place in college so that pre-service teachers are more prepared to teach students number sense strategies.

LeSage (2012) conducted a study where they had a select number of pre-service teachers take an additional course that focused on effective mathematical teaching strategies. A lot of the instruction taught teachers how to use concrete to abstract thinking with different mathematical skills. The study found the course to be effective if the pre-service teachers came into the course having some understanding of math concepts. If the pre-service teachers had very little knowledge, then the course did not make much difference because they would need more intense instruction themselves for a longer period of time. The previous research showed that teachers are lacking the mathematical knowledge. However, what was not stated, was the kind of knowledge teachers need to be prepared to teach students. In addition, the research showed that teachers are lacking the knowledge on how to teach mathematical skills to students. LeSage (2012) stated that more research could be done on math pedagogy to help prepare teachers on how to teach students the most effective way. The lack of teacher knowledge was not the only area needed for future research based on the compiled research studies analyzed in this literature review.

Future Research

From the research that was reviewed, the researchers shared areas in which further or future research could be done. Jordan et al. (2012) indicates more research could be done with the impact math development and instruction has on English language learners. Bryant et al. (2008) did research on tier two instruction but explain that further research of tier 3 instruction should take place. The last area of future research that was noted from the studies in this literature review was comparing memorizing versus understanding and explicit versus implicit teaching with mathematics. Bryant et al. (2008) conducted a study on the explicit instruction that is supported in math textbooks for kindergarten through second grade. They only looked at the main lessons of the textbook that involved numbers and operations. Bryant et al. (2012) looked at four textbooks, three of them being traditional textbooks and the other one was a reform-based textbook. Bryant et al. (2012) found that explicit instruction was not well supported in any of the textbooks they reviewed. The researchers state that components of mathematic textbooks could be researched again in the future to see if textbooks have changed their content as more research comes out in support of explicit instruction.

Memorizing Versus Understanding

One main disagreement that appeared in the research was how students should learn math, especially basic math facts. Gillum (2014) calls one method the traditional standpoint. This traditional standpoint includes memorizing algorithms and facts to be restated fluently. Geary, (2004) makes the point that memorizing a bunch of algorithms will not last in long-term memory for many students. Baroody (2006) calls the traditional standpoint as conventional wisdom. In the traditional view or conventional wisdom, students go through a developmental state when solving problems. Students often start solving problems using their fingers, then students tend to move towards using other problems they know to solve an unknown fact, and eventually students should be able to recall the facts (Reys et al., 1998). According to Baroody (2006), the opposite of the conventional wisdom is the number sense view. The number sense view's goal is for students to learn many facts so that they are stored in long-term memory. Cowan et al. (2011) states that the number sense view does not mean that students know every simple addition and subtraction problem. Instead, students should be able to use the facts that they have stored in their long-term memory to be able to solve other problems. For example, a student might know that 3 + 4 = 7 because they have 3 + 3 = 6 stored in his or her long-term memory.

Kilpatrick et al. (2002) also talk about the disagreement of memorizing versus understanding mathematics. Kilpatrick et al. (2002) references memorization versus understanding due to the math reform of standards from the National Council of Teachers of Mathematics in 1989. Kilpatrick et al. (2002) informs that only memorizing or only understanding is not enough. When students learn to only memorize, they will not be able to store enough in their long-term memory and will eventually make mistakes. When students only understand, they may not have the automaticity that is wanted. Instead, Kilpatrick et al. (2002) shares that students need more than only understanding and memorizing. Kilpatrick et al. (2002) states that for students to be well-rounded in mathematics, they are math proficient. The researchers created five branches of math proficiency that all students should be exposed to and practice in their math education. These five branches were explained earlier in this literature review in the instructional strategies section.

Another different viewpoint came from Courtney-Clarke and Wessels (2014) and Woods et al. (2018). They state that number sense should be directed through students' strategies and less focus on teacher direction. Students should be challenged with higher leveled thinking and asked to share their understanding. Woods et al. (2018) discussed that students should be challenged to share their knowledge to promote a strong understanding of number sense. This is where future research could occur about implicit instruction that promotes students' discovery-based learning from Courtney-Clarke and Wessels (2014) and Woods et al.'s (2018) research.

Memorizing, learning to understand, explicit instruction, and implicit instruction may connect together in stages of instruction, yet have some differences. Given the information that is shared in this literature review, it supports the need for further research in how to incorporate all of these into teachers' instruction. A variety of variables was brought up in the research that may impact students' mathematical development. A common variable that was discussed in several studies, was the socio-economic status of students.

Socio-Economic Effects

Courtney-Clark and Wessels (2014) and Andrews and Sayers (2015), look at the impact home life has on students' number sense development. According to Courtney-Clark and Wessels (2014), when children are not exposed to mathematical conversations at home before coming to school, building number sense will be a little more challenging for those students. With the proper instruction, students in low SES can build their mathematical knowledge, but it may take a little longer than they would if they would have come to school with that prior knowledge (Andrews and Sayers, 2015). Courtney-Clark and Wessels (2014), state that when students do not come to school with prior knowledge, then explicit instruction is even more important. Andrews and Sayers (2015) and Courtney-Clark and Wessels (2014), share that students from high income families and parents with high education are more likely to come to school with some mathematical knowledge. Andrews and Sayers (2015) state that students in a high socioeconomic status household are five times more likely to know comparison problems than a student who is in a low SES household. Kilpatrick et al. (2002) explained a study that was done with low-income first through sixth graders. These students were divided into two rooms with different types of instruction. One room's math instructional method was for students to focus on computing and memorizing algorithms. The other room's math instructional method focused on understanding math concepts and connecting problems to real world scenarios. In this room, students would discuss ideas and solutions with one another. After the two-year study, it was found that students in the room where understanding, discussions, and problem-resolutions took place, the better their scores were than the students in the computation focused room. This study showed that students' socio-economic status did not impact students' results. The way the students obtained instruction made a difference in students' mathematical knowledge. Morgan et al.'s study (2009) looked at whether a child's reading level impacted his or her math ability. In the study, the researchers controlled social class, age, race, and gender of the students. It was found that sociodemographic can predict a child's math development after kindergarten. In conclusion to these studies, it is undetermined whether socio-economic status impacts students' math growth and achievement due to the mixed results. Literacy is another variable found in the research that may impact students' mathematical development.

Literacy Connection

Reading instruction has changed drastically in education in the last twenty years. Clarke et al. (2011) states the importance of early literacy instruction and students' knowledge of phonemic awareness to impact a child's long-term reading ability. Response to Intervention or RTI is beginning to take place in schools to try to start interventions with students that struggle in reading. The same is just now coming to realization of mathematics. It has been found that number sense is the key to mathematical knowledge (Clarke et al., 2011). Fuchs et al. (2004) found that good readers who struggle with math, are more likely to react different to instruction than students who struggle with both reading and math.

Morgan et al.'s study (2009) looked at whether a child's reading level impacted his or her math ability. It was found that students struggling in reading had no impact on a student's math growth. The study found that sixty percent of students that struggled with math at the fall and spring testing times also struggled with reading. In conclusion, it is common that a child who struggles in reading will also struggle in math. Based on Morgan et al.'s study (2009) results, reading ability doesn't predict a child's math ability.

Cowan et al. (2011) held a study with second and third graders to see if literacy impacted math development and the way students solved basic addition and subtraction problems. It was found that reading did not impact students' results very much in regard to their math ability. Cowan et al. (2011) state that even though reading did not impact students' math ability at a young age, that does not mean it will not impact students as they get older, due to the fact that the skills of reading and math change. Although Cowan et al.'s (2011) study did not show correlation of math and literacy skills, Cowan et al. (2011) referenced other resources that did find a math and literacy connection. Durand et al. (2005) found a connection between math and verbal skills in their study held with seven to ten-year olds. The stronger verbal skills students had, the more it helped them with verbalizing math and understanding math vocabulary. Overall, the research presented in this literature review show mixed results as to whether literacy skills are connected to students' mathematical skills. Kilpatrick et al. (2002) points out that all students can learn math. In the past, and even some people today, state that groups of students may not become proficient in math. It may be at a different pace, but with the right interventions and integrated branches of instruction, Kilpatrick et al. (2002) clarifies that any student can learn math.

Conclusion

In conclusion of this literature review, research shows that number sense does effect students' achievement and development of mathematics. There are many different definitions of number sense but in this literature review it was defined as students understanding what numbers mean, how they relate to one another, and applying their number knowledge to other mathematical areas and application problems. The amount of math knowledge students came into kindergarten with showed a big difference in the success of students' short-term math development. Research has shown that learning about math and counting is very important in preschool for students' success in kindergarten. Extensive research showed success for explicit instruction to help struggling students or students who did not have any prior mathematical knowledge. Explicit instruction includes other instruction strategies that has shown to be beneficial to students' learning. Some of these strategies includes: Teacher modeling, gradual release, pre-teaching, review of previously learned content, instant and specific feedback, and positive error correction. Additional strategies and programs or frameworks that were created to support students' understanding of number sense was also included. Research agreed that the knowledge of mathematics that teachers have can impact students' math success. Teachers need to have the math knowledge and the understanding of concepts to be able to effectively teach math to students. Sources show a lack of teaching these skills in teacher preparation programs. A controversial area found in the research involved whether students should memorize or understand the meaning of problems. Researchers indicate that students will eventually memorize material after they understand and often use the strategies. Another researcher stated that neither memorizing nor understanding is enough and more characteristics, like application should also be included in instruction. Therefore, an area of further research in the memorization and understanding of mathematics could be done. Implicit or discovery-based instruction was brought up in the research several times to oppose explicit instruction. The majority of the research supports explicit instruction, but some researchers point out the importance of allowing students to figure out strategies on their own and to discuss his or her findings. The impact of socio-economic status and literacy showed mixed findings based on the literature that was reviewed. One source stated that just like literacy, any student can learn mathematics if the student is receiving high quality instruction that he or she needs.

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