The Effects of GeoGebra on Student Achievements, Critical Thinking/Problem-Solving Skills, and Engagement/Motivation in High School Mathematics

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Abstract

This research project explores the overall effects that GeoGebra has on students' learning in mathematics. More specifically, this research demonstrates the effects GeoGebra has on student achievement scores, students' critical thinking/problem-solving skills, and students' engagement/motivation. In this research, two groups of high school geometry students from two different years were used to determine students' achievements and understanding. Students in 2018 used traditional teaching practices while students in 2019 used GeoGebra as a means of learning mathematics. The scores were taken from teacher made assessments and reported as averages from both years. In addition, observations and a survey were used to determine how GeoGebra affected students' critical thinking/problem-solving skills and their engagement/motivation in the classroom. Overall, GeoGebra showed an increase in student achievement scores, critical thinking/problem – solving skills, and engagement/motivation. Therefore, GeoGebra plays a major role in the teaching and learning of high school mathematics. This research project also aims to give teachers the confidence in how to use GeoGebra in their own classrooms to further develop students in the areas of student achievements, critical thinking/problem – solving, and engagement/motivation.

The Effects of GeoGebra on Student Achievements, Critical Thinking/Problem-Solving Skills, and Engagement/Motivation in High School Mathematics

Mathematics is widely known as one of the most difficult subjects to learn. Many students and parents will often say, “I am not very good at math.” In addition, mathematics is often associated as one of the most boring subjects that everyone is required to take in school. It is often the subject described as “When am I ever going to use this?” It is no doubt that students’ achievement scores, motivation levels, and thinking skills are low when it comes to mathematics when they cannot find a use for mathematics or they believe that cannot learn the subject.

Given the fact that mathematics achievement scores, engagement, and critical thinking skills are low, it is imperative that the teaching and learning of mathematics must change. One of the ways that this can be done is through the teaching and learning of 21^{st} – century skills from the adoption of the Common Core State Standards. One of the 21^{st} – century skills that students need to learn is how to use technology proficiently and appropriately. Therefore, it is important for teachers in all content areas to find ways to incorporate technology into their classrooms. In mathematics, one of the most impactful technology resources is called GeoGebra which combines algebra and geometry into one dynamic software. There are other technology resources available for teaching mathematics, but this research project will strictly focus on GeoGebra. When technology is incorporated into the classroom, it will influence students and their learning.

Therefore, this research project looks to determine the effects that GeoGebra has on high school students. The research questions being explored in this research are:

1. What are the effects of GeoGebra on students’ achievements and understanding in learning high school mathematics?

2. What are the effects of GeoGebra on students’ critical thinking and problem-solving skills in high school mathematics?
3. What are the effects of GeoGebra on students’ engagement and motivation in high school mathematics?

Review of the Literature

Teaching 21st – Century Skills

“Today, education has a complex task to prepare students for the future that we can hardly predict” (Budinski, 2017, p. 3). As the world continues to evolve, so does the job and career field. Since jobs, businesses, and careers are changing, schools and education should also be changing to help prepare students. According to Umugiraneza, Bansilal, and North (2018), “teachers are urged to develop learners with relevant modern skills that match the needs of our changing world” (p. 1). In the past, teachers were required to teach students basic formulaic knowledge and skills in hopes they would use what they had learned when they graduated high school (Urbani et al., 2017). However, teachers, principals, parents, and researchers all agree that we need to shift from the teaching of basic facts and definitions to teaching students how to think, work together as a team, and be able to proficiently use technology which are more commonly known as the 21st – century skills.

One of the ways that teachers move past the teaching of basic facts and definitions is by teaching students 21st – century skills. According to Budinski (2017), students need to learn how to communicate, work in a team and become responsible citizens rather than just learning facts. One of the ways this change has come about was from the adoption of the Common Core State Standards (CCSS) in 2009. According to Urbani et al. (2017), “current educational policy, such as the Common Core State Standards, represents a shift away from rote learning and memorization of facts to the development of the 21st – century skills of creativity, critical thinking, communication, collaboration, and information, media, and technology skills” (p. 27). The Common Core State Standards put an emphasis on both English language arts and mathematics, but the standards can also be transferred towards other subjects like science and history and many others. Urbani et al. (2017) goes on to explain that when teachers read the standards, the elements of creativity, critical thinking, collaboration, and the use of technology are all explicitly stated within the literacy and mathematics standards. Therefore, from the Common Core State Standards, teachers are now required to go beyond the learning of basic facts and definitions to instead teach students how to critically think, communicate with each other, and use technology appropriately which are also known as the 21st – century skills.

One of the more growing concerns within teaching the 21st – century skills is teaching students how to use technology and how to use technology appropriately in different school settings and classes. Recently, school districts have started going one – to – one where every student and teacher is either given a laptop or a tablet that may be used in the classroom. Since most schools are going one – to – one, teachers need to learn what technology programs will work best for their classes and classrooms. Once teachers determine technology resources that work best for them and their students, the teachers will need to learn how to use them so that they may be properly implemented.

GeoGebra

Since technology is an important aspect of the 21st – century skills, it is important to find technology resources that can be used within different content areas. In mathematics, one of the most impactful technology resources is called GeoGebra. According to Hähkiöniemi and Leppäaho (2012), GeoGebra is a technology program that connects geometry, algebra, and calculus into one dynamic software. This dynamic software can be used either on a computer, tablet, or a phone. In addition, GeoGebra is available in both a downloadable version or a “web” version that are both free for anyone to use (Kovács, Recio, & Vélez, 2018). According to the same researchers, the downloadable version of GeoGebra may have faster speeds when students and teachers are creating their own applets, but the “web” version is quicker to access and use right away (Kovács et al., 2018). Depending on the technology devices and preferences by the teacher and students, one version may work better for them.

GeoGebra also allows teachers to either create their own applets or teachers can use what other teachers have already created. For example, GeoGebra has millions of free applets on their website (www.geogebra.org) that other teachers have already created, uploaded to their account, and shared publicly to the rest of the world (Kovács et al., 2018). Teachers who are new to GeoGebra may find it easiest to use what other teachers have already created because of the complexity of the dynamic software. Once teachers and students start to become comfortable in their abilities with GeoGebra, they can create their own applets that can be shared with their colleagues or to GeoGebra’s wiki page (Andresen & Misfeldt, 2010). At the same time, most teachers may want to create or find GeoGebra applets that are already created before each lesson for their students to use (Hähkiöniemi & Leppäaho, 2012). Andresen and Misfeldt (2010) agree that “teachers must be able to use the tool for problem solving and modelling him/herself” (p. 170). The best way for teachers to feel confident in their abilities to use GeoGebra is by having prepared GeoGebra applets before they are ready to teach the lesson.

Finally, since GeoGebra is one of the most dynamic mathematical software’s available, there are several ways that it can be used by teachers in the classroom. According to Budinski (2017), the most common ways to use GeoGebra is through demonstration, exploration or modelling, experimental work and creation. Most researchers have found that the best way to use GeoGebra is through investigation and exploration methods. For example, researchers like Hähkiöniemi and Leppäaho (2012) used GeoGebra to teach prospective math teachers how to utilize technology in the classroom. These researchers had the prospective math teachers investigate the relationship between a graph and the equation of the graph. Hähkiöniemi and Leppäaho (2012) both stated that the students using GeoGebra "can investigate connections by dragging points on a graph to see how the corresponding algebraic formula changes dynamically" (p. 46).

Budinski (2017) also used an investigative approach when using GeoGebra. He went on to explain that "GeoGebra's options provide quick exploration of basic mathematical concepts which are integrated in complex calculations and constructions" (Budinski, 2017, p. 149). In GeoGebra, teachers can create sliders that allow students to drag them and makes the graph or visual image move in some way. Students are also able to drag points to change the shape of images to explore the mathematical concepts that are taking place. Using GeoGebra as an exploration and investigation tool allows teachers to teach differently and it also allows students to think differently.

GeoGebra on Critical Thinking

Traditional teaching vs. facilitator. Mathematics and mathematics teachers have always been under fire about their teaching practices and how to teach mathematics in a way that keeps students engaged, make them use problem solving and critical thinking skills, and to show how mathematics can be useful after high school. In fact, one of the most common questions for students to ask a mathematics teacher is, "When are we ever going to use this?" This question forces mathematics teachers to reflect on their teaching styles and practices. Often, mathematics teachers can be found using the traditional role of teaching through lecture. Teachers tell the students what they need to know, the students do some example problems in class, and then they get a homework assignment that they do for the next day. According to Martinez and McGrath (2014), “education leaders, principals, teachers, policy makers and researchers increasingly agree that the traditional role of the teacher must change” (p. 41). Instead of acting as the authority figure in the classroom telling students what they need to know, mathematics teachers need to start taking on a different role where they act as facilitators of knowledge.

Martinez and McGrath (2014) agree that teachers should instead act as facilitators of learning rather than use the traditional style of lecturing. Both researchers stated, “as facilitators, teachers become learning strategists who constantly plan ways to enable students to master complex content knowledge and develop their critical thinking, problem-solving, communication, and collaboration skills” (Martinez & McGrath, 2014, p. 42). Instead of lecturing students, the teachers are creating activities and assignments for students that make them think critically and make them practice their communication and collaboration skills with each other. Finally, when teachers act as facilitators, they force their students to take more responsibility for their own learning (Martinez & McGrath, 2014). Students are no longer memorizing facts and definitions to regurgitate back on a test. Instead, students are crafting their own knowledge through their own research, critical thinking, and problem-solving skills.

Teacher training. One of the major issues of reversing the role of the teaching from traditional role to more as a facilitator, is that teachers do not know how to teach this way. Teachers usually go back to the ways that they were taught in school which was usually done through lecturing. For teachers to become facilitators to get students to critically think, they must have the proper training through workshops and professional development opportunities. In addition, teachers and students in most schools have all been equipped with computers or a tablet that they do not know how or when to use either. Researchers like Khalil, Sultana, and Khalil. (2017) argue that teachers need to have the proper skills and competencies to proficiently integrate technology. In fact, most teachers will try to use technology and say that they are now teaching as facilitators. According to Martinez and McGrath (2014), “teacher roles won’t change by simply booting up and connecting to the internet” (p. 42). Teachers will need to learn how to use technology in a way that helps them act as facilitators. Even though it will not be easy, “teaching can be learned, and teachers should be a part of scholarly community” (Khalil et al., 2017, p. 84).

Given the fact that teachers need to learn how to teach as facilitators that make students critically think and how to use technology effectively, teachers need to be given the chance to learn through professional development opportunities and teacher training sessions. According to Andresen and Misfeldt (2010), they had the privilege of training teachers how to use GeoGebra through a teacher training session. The researchers met with a private school (K- fifth grade) with 12 teachers. The training session lasted a total of thirty hours and the training sessions were aimed at training teachers how to use GeoGebra through the schools’ shared vision of teaching mathematics. During the workshop, teachers were able to play with GeoGebra and had informal talks and discussions about teaching and GeoGebra. By the end of the workshop, teachers thought it was a productive amount of time where they were able to discuss both theory and practice. Therefore, for teachers to effectively use technology in a way that makes them facilitators of knowledge, teachers must attend trainings, workshops, and professional development opportunities.

Facilitating with GeoGebra. But how is GeoGebra best used? Researchers like Bozkurt and Ruthven (2017) had the privilege of examining how a teacher (Chris) was able to use GeoGebra effectively in his classroom. Chris used GeoGebra in a way that allowed his students to explore the mathematical concepts. He prepared GeoGebra files beforehand for students to use in each lesson because GeoGebra is a very complex software that can be difficult to understand how to use. During the very first lesson, Chris demonstrated how to use GeoGebra to his students. He showed the students how to open GeoGebra, where to find the applets that he had already created, where the Toolbar and Toolbox was located, and some of the basic functions built into the software. He even showed the students how to drag objects in GeoGebra to show the students how to manipulate shapes and graphs.

Once he had demonstrated how to use GeoGebra to his students, he could then go into the lessons that he prepared which was on the unit of transformations. The researchers noted that Chris's lessons could be broken down into three phases. The first phase was where he introduced the lesson to the entire class. During this phase, Chris would demonstrate the GeoGebra applet on his interactive whiteboard and he would give the students any information on what he expected his students to do. During the second phase, students went on their computers and worked with the prepared files that Chris had made. The students would use the GeoGebra applets to explore the mathematical concepts of transformations while Chris would walk around the classroom to guide the students in what they were doing. During this phase, Chris encouraged his students to work in groups and to make conjectures about what they thought was true. Once they wrote down their conjectures, Chris had the students test them. The last phase of each lesson was where Chris brought the class back together as a whole. Chris wanted to connect each groups' independent work and connect their conjectures to the main objectives of the lesson. He usually did this by having students share their experiences and conjectures with the rest of the class. Through this method, Chris found that students were able to develop their critical thinking skills, problem-solving skills, and promote communication and collaboration using GeoGebra.

GeoGebra on Engagement

Students are all created differently. Some students will enjoy learning mathematics while other students will not. Several researchers found that working with GeoGebra helped motivate students learning of mathematics. First, Chehlarova and Chehlarova (2014), found that students who worked with the dynamic software GeoGebra were more engaged and motivated in learning mathematical theories and applications. In fact, Chehlarova and Chehlarova (2014) mainly focused on students that were interested in learning about art. The researchers took regular pictures of students to help motivate students in learning mathematics. One method the researchers used was attaching the pictures to the graph of a function. The students could then use a slider to move the picture along the graph of the function. In addition, the researchers had students use pictures to explain the concept of transformations. The students were able to dilate the picture to make it bigger or smaller along with translating, rotating, or reflecting the picture. By the end of the experience, the researchers concluded that the students who were interested in art where more motivated when they could use GeoGebra to explain mathematical concepts. Furthermore, the researchers found that these students usually went above and beyond the requirements of most students when they could use technology like GeoGebra (Chehlarova & Chehlarova, 2014).

López (2011) also found students were more motivated in learning mathematics when they could use GeoGebra. The researcher used GeoGebra to explain the points of concurrency

(Circumcenter, Incenter, Centroid, and Orthocenter) in geometry lessons. At the end of the experience, the researcher had the students fill out a survey to get the students perspective on GeoGebra. From the survey, the researcher discovered that 88% of the students did not find working with GeoGebra to be boring (López, 2011). In addition, the students had said that they were interested in GeoGebra because it "helped resolve doubts about mathematical concepts that were already given" (López, 2011, p. 188).

Finally, Zengin and Tatar (2017) also found GeoGebra to help motivate students in learning mathematics. The researchers wanted to determine whether GeoGebra would increase students' attention and interest in learning mathematics. To determine this information, the researchers collected data through qualitative analysis methods of questionnaires. Two of seven students stated that GeoGebra increased their interest and their motivation in learning mathematics. The students had said, "the combination of dynamic mathematics software and cooperative learning increased my interest and motivation" (Zengin & Tatar, 2017, p. 80). At the same time, another student had stated, "it is an enjoyable learning technique in which we listen and implement without getting bored" (Zengin & Tatar, 2017, p. 80). Given the amount of research from the articles, it can be concluded that using GeoGebra can increase engagement and motivation in learning mathematics.

GeoGebra on Student Achievement

There have been researchers that have also reported GeoGebra having positive effects on students' achievements and understanding in mathematics. First, Zengin and Tatar (2017) both had positive findings on students' abilities and achievements when they used GeoGebra to teach quadratic functions. The researchers had 61 high school students participate in the study. In total, 44 of the students were in the experimental group where students could use GeoGebra while 17 of the students were placed in the control group in which traditional learning methods were used. The researchers used both quantitative and qualitative approaches to collect data on student achievements. To collect quantitative data, the researchers had both the experimental and control groups take a pre-test and a post-test that were identical. The students who were in the experimental group and the control group scored very similar to each other in the pre-test. The average score for the experimental group was 18.92 and the average score of the control group was 18.03 (Zengin & Tatar, 2017). However, the results were much different for the post test. The experimental group had an average score of 22.26 while the average score of the control group was 14.29 (Zengin & Tatar, 2017). Therefore, the researchers concluded that students who learned through cooperative learning methods with the support of GeoGebra increased student achievements.

At the same time, Zengin and Tatar (2017) also had students fill out questionnaires to get the students' perspective on using GeoGebra in relationship to their achievements and understanding. From the questionnaire, both researchers were able to conclude that GeoGebra helped promote better understanding, by enabling students to visualize the course which increased retention. In fact, 15 students stated in the questionnaire that GeoGebra helped them understand the mathematical concepts better (Zengin & Tatar, 2017). Fourteen students also said that the software helped them visualize the concept of quadratics and that the material was much more concrete (Zengin & Tatar, 2017). To help the students visualize the concepts of quadratic functions, they were given a GeoGebra applet that contained a quadratic function in standard form ($f(x) = ax^2 + bx + c$) and three sliders that changed the values of *a*, *b*, and *c*. Students were also given guided questions and asked to explain how each slider affected the graph of the parent function. When students moved the slider, they could visually see how each value effected the graph of a quadratic function. Finally, 13 students stated that GeoGebra helped the students retain the information because it was much more hands-on learning (Zengin & Tatar, 2017). Instead of the teacher telling the students exactly what each value did to the quadratic function, students got hands-on experiences by moving sliders to manipulate the quadratic functions.

Other researchers like Richardson and Koyunkaya (2017) also tried to determine the effects of GeoGebra on students' achievements and understanding. These researchers focused on students' understanding on the definitions of angles. To start, the researchers had three students share their definition of what they thought an angle was. The most common responses from the students were classifying angles based on degrees or the students would draw an angle, but they could not explain why their drawings were angles. All the did was point at the picture. After working with GeoGebra, the students' definitions of angles had changed. For example, Student 1 had two different definitions from the experience. The student's initial definition was drawing two lines and then saying it was an angle. After working with GeoGebra, the same student said that an angle is two-line segments that rotate. This was just one example of three that was reported. Once the researchers collected the data from the students, they concluded that students were able to improve the students' mathematical development when they were given the opportunity to manipulate objects in a dynamic software like GeoGebra.

Finally, Hutkemri and Akmar (2016) found GeoGebra to help student achievements. Both researchers decided to focus on students' conceptual knowledge and procedural knowledge to gage students understanding and achievements in calculus. To begin the study, the researchers first defined conceptual knowledge, procedural knowledge, and academic achievement. First, conceptual knowledge was defined as "knowledge that is rich with relations and knowledge about concepts or principles" (Hutkemri & Akmar, 2016, p. 155). On the other hand, Hutkemri and Akmar (2016) defined procedural knowledge as “understanding that involves numerous facts and definitions” (p. 155). Finally, academic achievement was defined as the students’ ability to learn subjects at school. After defining these three areas, the researchers wanted to determine whether GeoGebra was effective in helping students in their conceptual, procedural, and academic knowledge.

During Hutkemri and Akmar (2016) study, the researchers had a total of 284 students. Of those students, 136 were placed in the experimental group where they would use GeoGebra to learn the concept of limits while 148 students were placed in the control group and were using traditional styles of teaching. The researchers found that students who used GeoGebra showed a stronger increase in students’ conceptual and procedural knowledge which affected the students’ academic achievements. It was also noted that the students who used GeoGebra obtained more conceptual knowledge while the students who received the traditional style of teaching received more procedural knowledge. However, both conceptual and procedural knowledge is important for students to gain better understanding in their academic achievements. Therefore, from the researchers, it can be noted that GeoGebra helped students in the achievements and understanding of mathematics.

Methods

Participants

For this research project, geometry students from the second semester of the 2018 school year and the second semester of 2019 school year were used to collect data. Therefore, two different groups of students were used to collect data. The students in 2018 were never exposed to the dynamic software of GeoGebra. This group of students will be considered the control group. In 2019, the students used GeoGebra for several different lessons. Therefore, the 2019 group of students will be considered the experimental group.

At the same time, in this school district, the geometry class has a mix of different students and abilities. Geometry is technically a sophomore level class at the high school, but depending on the abilities of the students, they can take geometry when they are either a freshman, sophomore, junior, or senior. Students who take geometry their freshman year have skipped their eighth-grade year of mathematics and went straight into algebra 1 in eighth grade. Students who take geometry their sophomore year are on the normal track of mathematics. Finally, junior and senior students are ones that usually struggle with mathematics, are retaking the class over again, or they are in the English language learners (ELL) program or special education department.

In the control group, there were a total of 67 students that took geometry. Of those 67 students, 39 were males and 28 students were females. In addition, there were 24 freshman, 42 sophomores, one junior, and no seniors. Finally, three students were in the special education department and none of the students were in the ELL program. On the other hand, the experimental group of students consisted of 70 total students. Of those students, 35 students were males and 35 students were females. In addition, 31 students were freshman, 36 students were sophomores, two were juniors, and there was one senior. Finally, there were three special education students and four students were in the ELL program. Overall, the total number of students were about the same between the two groups except there were more freshman and ELL students in the experimental group than the control group.

Learning to Use GeoGebra

Before data could be collected, it was required to go through specific trainings on how to use GeoGebra before it could be implemented into any lessons. As stated by the literature review by researchers like Khalil et al. (2017) and Andresen and Misfeldt (2010), teachers need to learn how to use GeoGebra in a way that helps the teacher become more of the facilitator rather than using lecture to teach. For this research project, pre-made GeoGebra applets that were created by other teachers were used at first by the teacher and students. However, as time progressed, GeoGebra applets were created by the teacher that were structured to fit closer to this geometry class. Geometry can be taught and explained differently depending on the teacher, resources, and textbook that is given to them. Therefore, it was more beneficial to have GeoGebra applets that were created by the teacher to implement into this research project.

When learning how to create GeoGebra applets, it was best to use what other teachers had already created. GeoGebra applets made by other teachers were downloaded from GeoGebra's website to the computer and then recreated from scratch to learn how to use the dynamic software. Two of the most impactful pieces of GeoGebra that were used in all of the GeoGebra applets were the slider and the check boxes. The slider allows students and teachers to slide a point across a segment which would then move the shape, points, and lines, in the applet. In addition, the check boxes allowed students to view or hide objects by just the click of the button.

After learning how to use GeoGebra, the students were then able to use GeoGebra in the classroom. The teacher used the three-phase system like Chris used to incorporate GeoGebra into the geometry lessons (Bozkurt and Ruthven, 2017). GeoGebra applets were created before the lessons and then they were uploaded to OneNote Class Notebook. OneNote was were the students could find online notes, videos, assignments, and other resources like GeoGebra for geometry. During class, the students were given an introduction of the lesson. The objectives were stated, and the students were given basic instructions of what to do with the GeoGebra applets from the teacher as they were displayed on the interactive whiteboard. After the instructions, students went to their laptops and used the GeoGebra applets to come up with their own thoughts and conjectures from what they observed. Students were encouraged to work in pairs or small groups to have conversations and to collaborate with each other. Once students were given the chance to come up with their own conjectures, students came back together as a whole class to discuss what they saw and what they thought were the important aspects of each GeoGebra applet. At the same time, the whole class discussion was a way to clarify any misconceptions the students experienced from the GeoGebra applets.

Data Collection

By the end of the experience, a mixed methods approach of both quantitative and qualitative data was collected for this research. Teacher made tests and quizzes were used to collect quantitative data on student achievements and understanding. Both the experimental and control groups were given the same tests and quizzes which were worth the same amount of points. There were no altercations to any of the tests from the two years. At the same time, the students' scores were taken as an average. The averages were then compared between the two groups to determine if GeoGebra helped raise students' achievement scores and their understanding of mathematical concepts in high school geometry.

Next, observations and a survey were used to collect data on students' critical thinking/problem solving skills and engagement/motivation. Observations were mostly used to collect data on students' critical thinking and problem-solving skills. Participant observation was used because the researcher was participating in the activities that were being studied (Mills, 2018). For example, the researcher was observing the GeoGebra activities that were taking place and the researcher was also engaging in the activities with the participants. The survey was mostly used to collect data on students' opinions about GeoGebra and to determine students' engagement and motivation levels in learning mathematics through technology rather than traditional roles like lecture.

Results

Data Analysis

Data was collected to determine whether GeoGebra helped students in their understanding of mathematical concepts. The experimental group was the group of students who used GeoGebra while the control group used traditional learning methods like lecture and were not given the opportunity to use GeoGebra. GeoGebra was used in a total of five units for this research project. Table 1 below shows the average scores from the experimental group and the control group. The average represents the expected value of what a randomly chosen student would receive on the assessment. In addition, the last column of Table 1 represents the differences of the average percentages. A positive number represents the students who used GeoGebra had a higher average percentage than the students who did not use GeoGebra while a negative number represents the students who did not use GeoGebra had an overall higher average on the assessment.

Table 1

Experimental Group and Control Group Mean

Assessment	Experimental mean	Control mean	Difference of mean
Quiz: Parallelograms	79.25	79.30	-0.05
Quiz: Quadrilaterals	73.96	75.06	-1.10
Test: Quadrilaterals	83.03	79.68	3.35
Quiz: Trigonometry	76.35	75.96	0.39

Quiz: Trigonometry	75.10	73.50	1.60
Test: Trigonometry	78.27	77.47	0.80
Quiz: Transformations	80.85	79.57	1.28
Test: Transformations	74.22	71.42	2.80
Test: Circles	75.65	75.35	0.30
Test: Perimeter and Area of Polygons	75.34	69.33	6.01

Note. Experimental group consisted of 70 students and the control group consisted of 67 students.

From Table 1, GeoGebra was used in a total of ten assessments in five units. During the first two assessments, students who did not use GeoGebra had an overall higher average than the students who used GeoGebra. However, the students using GeoGebra had a higher average on the final eight assessments. In addition, most of the assessment scores were within 3% of each other. The one exception was the assessment on perimeter and area of polygons where the experimental group had a 6% higher average than the students who did not use GeoGebra.

Table 2

Experimental Group and Control Group Standard Deviation

Assessment	Experimental group σ	Control group σ
Quiz: Parallelograms	6.09	5.66
Quiz: Quadrilaterals	5.13	5.08
Test: Quadrilaterals	3.31	3.36
Quiz: Trigonometry	10.38	8.87
Quiz: Trigonometry	6.66	7.28
Test: Trigonometry	10.09	9.19
Quiz: Transformations	3.48	3.70

Test: Transformations	5.38	6.36
Test: Circles	6.18	7.30
Test: Perimeter and Area of Polygons	4.46	4.81

Note. σ is the Standard Deviation of the population

Table 2 shows the standard deviation of the population of both the experimental group and the control group. Since all the students were used in both the experimental group and the control group, the standard deviation of the population was used rather than the sample standard deviation. If the standard deviation was a bigger number, then the data was spread out farther away from the average. On the other hand, if the standard deviation was smaller, then the data was closer to the average. For example, if the average percentage of a set of data was 75% and the standard deviation of the data was ten, that means that students were scoring 65% and 85% on their assessments when the average was taken. If the standard deviation was low, then students were scoring very close to the average. Therefore, it is better to have a lower standard deviation as the students were scoring closer to the average. From Table 2, the students who used GeoGebra had a lower standard deviation in six of the ten assessments.

Students also showed an increase in their critical thinking/problem-solving skills when they were using GeoGebra. The students who did not use GeoGebra were always told what they needed to know through lecture, then students would try a few problems, and then they would get a homework assignment with similar problems as the in-class examples. However, with the use of GeoGebra, students were now required to determine what they thought was happening on their own, with a partner, or in small groups. For example, one of the units that GeoGebra was used for was on transformations. Each year, students had to determine the rules for transformations so that they could reflect, rotate, translate, and/or dilate a figure on the coordinate plane. In the control group, students were just told what each rule was and then students had to apply that rule to each problem. Students in the experimental group were given a GeoGebra applet to develop their understanding of rotations. Students were given the link to this GeoGebra on their OneNote Class Notebook page and then they were required to determine the rules on their own by using the sliders and input boxes. When students entered information into the input boxes, GeoGebra would offer immediate feedback to the students on whether they got the correct answer or whether they needed to try again. This method of teaching was used for the experimental group in every lesson of the five units that were covered.

While students were working through these GeoGebra applets to determine their own conclusions and conjectures, teacher observations were used to collect data and then data was entered in a journal. Multiple occasions, students were working with a partner or two, asking each other questions, and digging deeper into the material. Students were writing their own conjectures into their notes, but more importantly, they were asking each other questions like, "Why does this work" or "How do you know?" In addition, from the journal, it was evident that students were being forced to explain the concepts to their partners. Students were teaching each other. Students were quick to ask each other questions and figure out the problem on their own rather than asking the teacher. Students took ownership of their learning. In the end, GeoGebra allowed students to investigate and explore mathematical concepts that made them think on their own and with each other rather than the teacher telling the students what they need to know.

Finally, students showed growth in their engagement/motivation. For this data, 51 students filled out a survey (Appendix A) to get some basic background on their learning interests and abilities along with what they liked and did not like about GeoGebra. Students were also encouraged to give their inputs on any changes that should be made on how GeoGebra should be used in the future or in ways that they liked using GeoGebra the best.

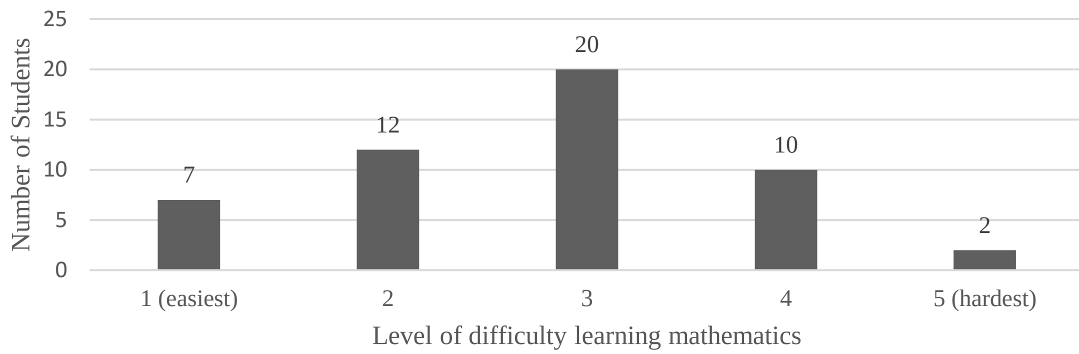

Figure 1. Students’ beliefs on the difficulty of learning mathematics.

From the survey, students were asked how easy or hard they thought it was to learn mathematics on a scale of one to five with one being the easiest. Figure 1 shows the number of students that clicked on each category. After analyzing the data, 13.7% of the students said mathematics was easy for them, 23.5% said it was slightly easy, 39.2% where in the middle, 19.6% said it was slightly hard, and finally 3.9% of the students said learning mathematics was extremely hard for them. This data was helpful to get the students perspectives and opinions about the difficulty of learning mathematics.

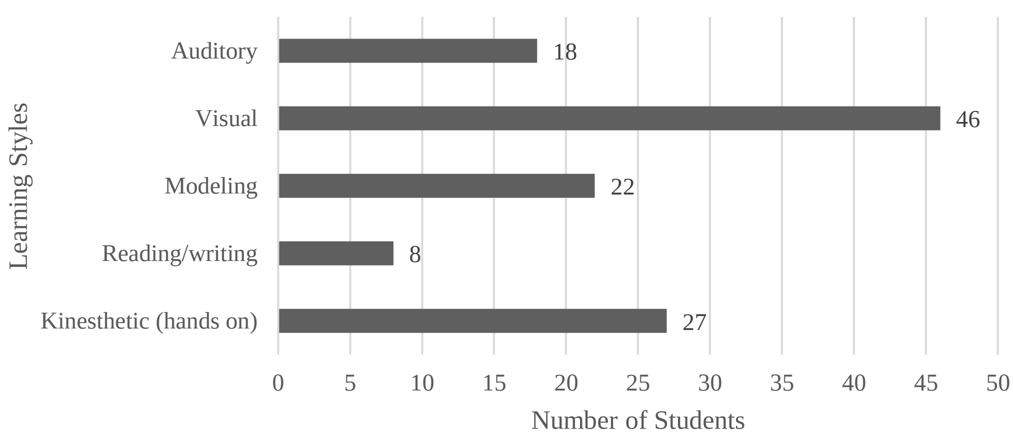

Figure 2. Students’ listing the ways they learn best.

Students were also encouraged to state how they learned best in the survey. Figure 2 shows how many students clicked in each category. Students were given the choices of auditory, visual, modeling, reading/writing, kinesthetic, or other. Students were able to click on multiple answers for this question. First, 35.3% of the students said that they learned best through auditory means. In other words, they liked to hear the information being presented to them. Second, 90.2% of the students that took the survey said that they learned best through visuals. Third, 43.1% said they liked modeling. Fourth, 15.7% of students said they would like to read/write to learn. Finally, 52.9% of students said they like kinesthetic learning meaning they like to work in a hands-on setting to learn. This data was very helpful because the two highest percentages were visual and kinesthetic learning. GeoGebra offers students a way to have both learning experiences. First, GeoGebra can be a hands-on experience if used correctly. In this research project, students were given the opportunity to investigate and explore the mathematical concepts on their Chromebooks. Students could engage with the GeoGebra applets and the material instead of passively listening. Second, GeoGebra was extremely visual. Students were using sliders and watching the shapes and applets change. GeoGebra brought mathematics to life and allowed students to visualize and make mathematics more concrete.

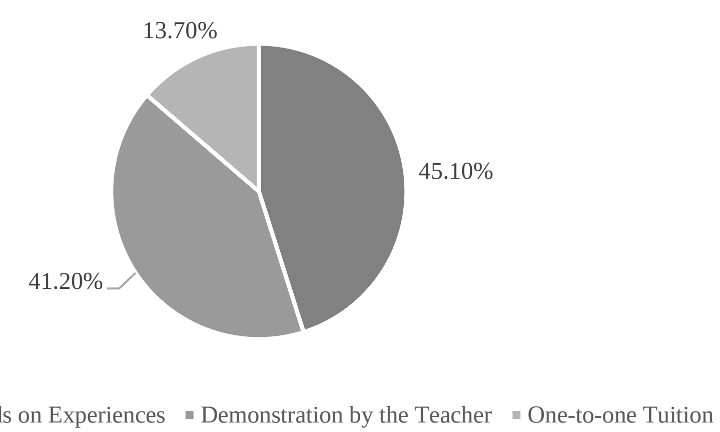

Figure 3. Students listing how they liked using GeoGebra best during in class activities.

GeoGebra was also used in a few different ways during the five units. At times, the teacher would just demonstrate what was happening in the GeoGebra applet. Other times, students would work hands-on and use exploration and investigation tactics. Finally, students would get to work one-on-one with the teacher while using GeoGebra. From the survey and shown in Figure 3, 41.2% of the students liked when the teacher just demonstrated and showed them what was happening. 13.7% of the students like one-on-one methods and 45.1% of the students liked using GeoGebra when they could explore during hands-on activities. This data was helpful to determine which way students liked to use GeoGebra the best. Most of the students enjoyed using GeoGebra as an exploration/investigation tool rather than being told what to do. See Figure 3 to see the percentages of students who clicked on each category.

Lastly, students were able to give their opinions through open ended questions of what they liked and did not like about GeoGebra. None of the students responded by saying that GeoGebra helped motivate them in mathematics. However, from the responses it was very evident that students were engaged while learning through GeoGebra. Many of the students liked how visual GeoGebra was and how they could work with GeoGebra in hands-on experiences. For example, one student responded by saying, “I liked to see the object move as the numbers moved and it gave me a visual in my head to remember it.” Another student had said, “I enjoy using GeoGebra because it helped me visualize the concepts we were learning.” In total, there were 30 students who stated they liked GeoGebra because it helped them visually understand the mathematical concepts or they liked GeoGebra because it was more of a hands-on learning experience for them. Even from the observational data and the teacher journal, it was evident that students were more engaged and motivated in working with GeoGebra. Students had said, “That is so cool” or “Wow, how does that work?” Students were engaged in the visual aspects of

GeoGebra and from the survey, 78.4% of the students wish that GeoGebra was used more throughout the entire school year.

Discussion

Summary of Major Findings

Students who used GeoGebra showed an increase of their achievements/understanding, their critical thinking/problem-solving skills, and engagement and motivation. First, students showed an increase in their levels of understanding and achievements. The students who used GeoGebra had a slight increase in their average assessment scores. Even though most of the averages were less than 3% increase, students were still scoring higher on average when they were using GeoGebra. Not only did the quantitative data from the assessments show that students gained better understanding and achievements, but students also posted from the survey that they thought GeoGebra helped their understanding. For example, one student stated from the survey, "I liked it (GeoGebra) because it showed me how and what we were doing make sense. It wasn't just the idea of following a formula because someone said it worked. GeoGebra showed me how that formula worked and made sense."

GeoGebra also showed improvements to students' critical thinking/problem solving skills. Students were not passively sitting and being told what to do. Instead, students made their own conjectures and conclusions from GeoGebra. Students shared their conjectures with each other, and they were asking questions like why and how does this work? Finally, the teaching style was changed. The teacher took on the role of facilitator in this research project. The students would determine their own conclusions and the classroom experience was built around discussions. If needed, the teacher would make any clarifications to the mathematical concepts during the whole class discussions.

Finally, students showed an increase in their engagement and motivation in learning mathematics. Students enjoyed being able to investigate and explore the mathematical concepts on their own and in small group settings. At the same time, students liked how GeoGebra took abstract concepts and made them visual and more concrete. In this research project, students were exposed to sliders and checkboxes in each GeoGebra applet. The sliders allowed students to move angles, points, segments, etc. within each picture and the checkboxes allowed students to hide or show geometric constructions quickly. These tools allowed students to engage with the material in a visual representation and therefore engaged and motivated the students in the learning of mathematics.

Limitation of the Study

There were a few limitations of this study. First, the assessments that were used in this research project were created by the teacher. Therefore, these assessments were not valid or reliable (Mills, 2018). In other words, the assessments were not valid in the fact that they may not measure what they are supposed to measure, and/or the assessments were not reliable because they may not produce consistent results. Using standardized tests would be considered valid and reliable tests. Standardized tests were not used because the control group and the experimental group took two different standardized tests. The students who did not use GeoGebra took the old Iowa Assessments which were mostly multiple-choice questions that required students to fill in the bubble. The students who used GeoGebra were required to take ISASP (New Iowa Assessments) tests which required more critical thinking and problem-solving skills. Therefore, since the two groups used different types of standardized assessments, this research project used teacher made assessments instead to calculate student understanding and achievements.

Second, there were two major differences between the experimental group and the control group. First, in the experimental group, there were 31 freshmen compared to 24 freshmen in the control group. Therefore, there were more students with higher abilities in geometry in the experimental group where students used GeoGebra than in the control group where students did not use GeoGebra. Second, there were four students in the English language learners (ELL) program in the experimental group while there were none in the control group. Students in the ELL department tend to struggle more because of the language barrier between the mathematics teacher and the students. Therefore, these two groups could have skewed the results of the quantitative data that was collected to determine students' achievements and understanding of mathematical concepts.

Further Study

Finally, this study could have been done over a longer time period. The students who used GeoGebra were using it for only the second semester of GeoGebra. This came out to be five chapters that students were using GeoGebra. To further the study, it would be interesting to see students' achievements, engagement/motivation, and critical thinking/problem-solving skills develop over an entire year of using GeoGebra. In addition, by the fifth unit, students who used GeoGebra showed an average increase of 6% in their achievements and understanding of mathematical achievements. Would this continue? Would students continue to show a much higher average over a longer period or was this just the result for that unit?

Conclusion

From the literature, it is evident that mathematics teaching and learning needs to change. In addition to learning the mathematical concepts, students also need to learn 21st – century skills. Since students need to learn 21st – century skills, teachers must develop a new way of teaching. The new form of teaching takes the teacher out of the traditional role and makes them more as a facilitator of knowledge. One of most common ways to switch the role of any teacher is to use technology in an appropriate way.

One of the more impactful technology resources in mathematics is called GeoGebra which combines algebra and geometry into one dynamic software. From the literature review, there were several articles that were written that showed positive effects when mathematics teachers used GeoGebra in their classrooms. In this research, GeoGebra was used to explore the effects on student achievement scores, students critical thinking/problem-solving skills, and students' engagement/motivation in high school mathematics. Specifically, GeoGebra was used in a geometry classroom of 70 students. In the end, students who used GeoGebra showed an increase in all three areas.

First, students showed an increase in their achievements and understanding. Teacher made assessments were used to calculate the average scores and standard deviation of the population from students who did not use GeoGebra last year and average scores were calculated with the students who used GeoGebra this year. When comparing the two sets of data, it was concluded that students who used GeoGebra had a better understanding of the mathematical concepts as their average assessment scores were slightly higher. Furthermore, while comparing the standard deviations, the students who used GeoGebra were scoring closer to the average while the students who did not use GeoGebra scores were much more spread out.

Second, students developed a stronger sense of critical thinking and problem-solving skills. Students were given pre-made GeoGebra applets where the link was provided on their OneNote Class Notebook page. Students went to the GeoGebra link and used the sliders and checkboxes in each GeoGebra applet and wrote their own conjectures. While students were working with the GeoGebra applets, observations were used to collect data based on student conversations and their ability to write their own conjectures. Once students wrote their conjectures, a whole class discussion was used to clarify any misconceptions. From the data, it was evident that students did show an increase in their abilities to critically think and problem solve. Students were asking each other questions like why and how and comparing their responses with their partners.

Finally, students showed an increase in their engagement and motivation while using GeoGebra. To collect data in this area, a survey was used to determine the ways that students like to learn best. At the same time, the survey was used to determine why students liked or did not like using GeoGebra. From the survey, it was evident that students like to learn through hands-on experiences and through visuals especially in a mathematics class. GeoGebra offered the students a way to do both through exploration methods. Students used their Chromebooks in hands-on experiences to access GeoGebra and GeoGebra offered great visuals by using sliders and checkboxes in each applet.

Ultimately, it is imperative that the teaching and learning of mathematics changes. This research project showed how GeoGebra can raise students' achievements, critical thinking/problem-solving skills, and engagement/motivation in high school mathematics. However, this research project also shows how mathematics teachers can help prepare students outside of high school by developing their 21^{st} – century skills. Finally, this research project shows other mathematics teachers how GeoGebra can be used in their own classrooms that takes them from the traditional role of teaching to one that is a facilitator.

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Appendix A

1. What grade are you in?

   Choose an item.

2. On a scale of 1 to 5 (1 being the easiest), how easy/hard is learning mathematics for you?

   Choose an item.

3. In what ways do you learn best (check all that apply)

☐Auditory ☐Visual ☐Modeling ☐Reading/Writing ☐Hands-on

4. Do you think GeoGebra helped you understand the concepts better in mathematics?

   Choose an item.

5. Would you liked to have used GeoGebra more or less in learning mathematics?

   Choose an item.

6. For yourself, what was the best way you liked using GeoGebra?

   Choose an item.

7. What did you like or not like about GeoGebra?

   Click or tap here to enter text.

8. Do you have any suggestions about how GeoGebra could/should be used next year?

   Click or tap here to enter text.